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Gravitational dynamical dimensional reduction in Causal Set quantum gravity

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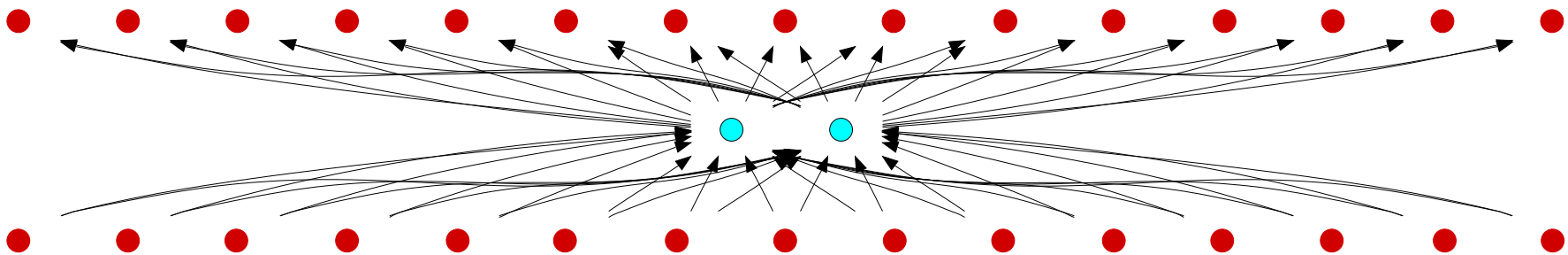

G. ’t Hooft, “Quantum gravity: a fundamental problem and some radical ideas", in *Recent developments in gravitation*, Cargèse Summer School Lectures 1978

- take lattice discretization seriously, as model for gravity
- metric tensor difficult to define on lattice, while respecting covariance
- simpler, invariant concept: causal ordering
- in discrete language, partial ordering
- this partial order *defines* the lattice
- consistency with causal ordering of continuum defines correspondence between discrete and continuum
- proposes that the early universe will emerge from a single discrete element, followed by a tree

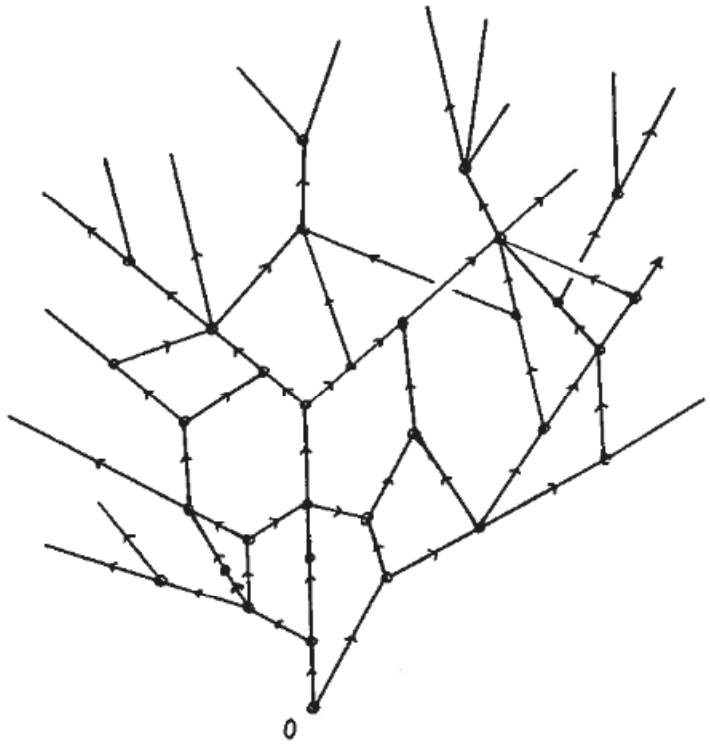

Fig. 10 The Cosmic Tree.

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Dynamics of causal sets
Introduction to discrete dynamics
Causal set dynamics via Sequential Growth
Transitive Percolation
Originary Percolation
Dimensional reduction
Summary and Conclusions

Plan of this Talk

1. Background
2. Kinematics of causal sets
3. Dynamics of causal sets
4. (Gravitational) dynamical dimensional reduction
5. Summary and Conclusions

Outline

1 Background

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3 Dynamics of causal sets

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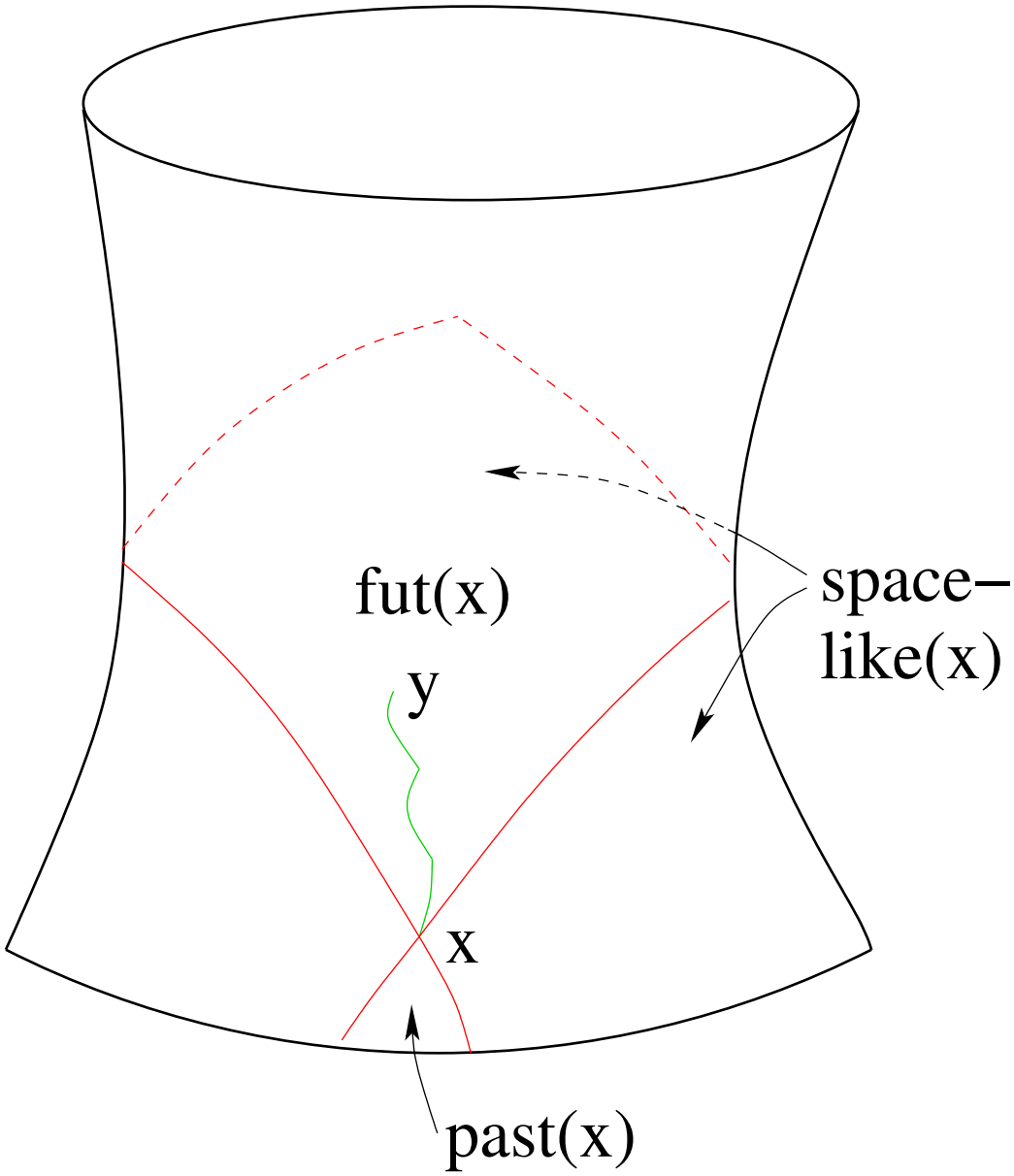
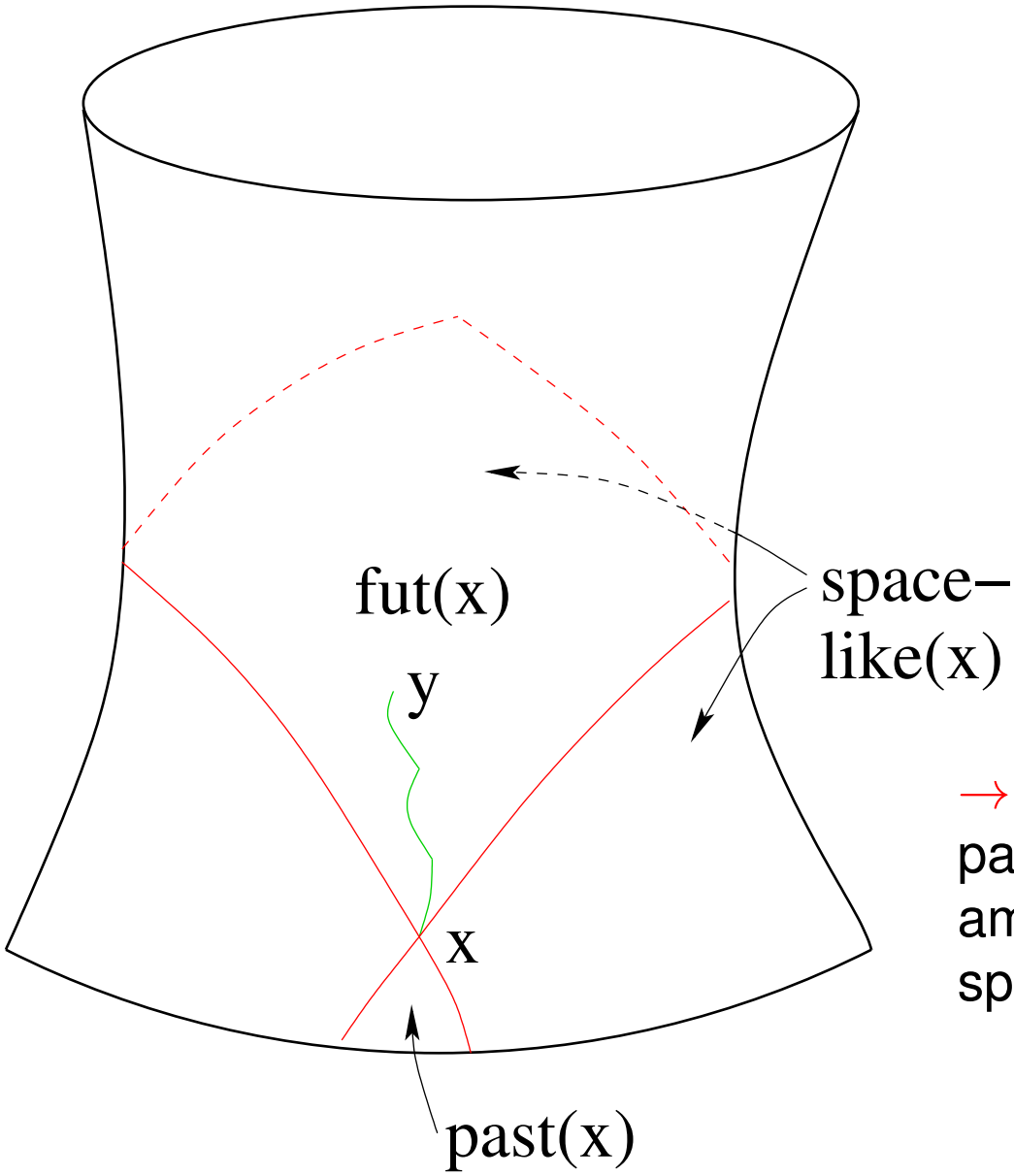
General Relativity and Causal Structure



- Differentiable spacetime manifold M
- Lorentzian (-+++) metric g
- Null directions of $g(x)$ separate M into three regions

General Relativity and Causal Structure



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→ Causal Structure
partial ordering $\prec$ among points in spacetime manifold M

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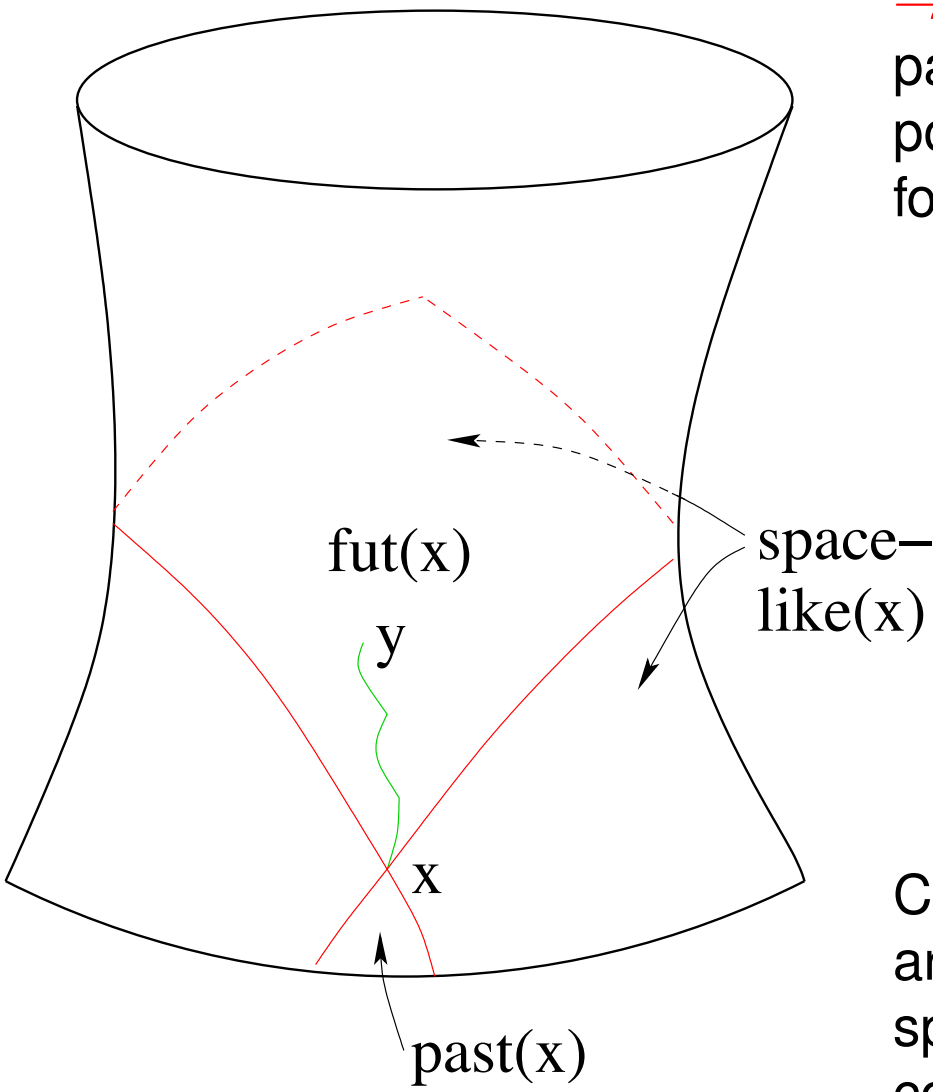

→ Causal Structure

partial ordering $\prec$ among points in spacetime manifold M

- A.A. Robb (1931)
- H. Reichenbach (1921)
- E. Zeeman (1964)
- S. Hawking (1966)
- Hawking, King, McCarthy (1976)
- D. Malament (1977)

Causal structure of past and future distinguishing spacetime determines the conformal metric, topology, and differential structure

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Fundamental Discreteness

Problems with the continuum:

- Divergences in Quantum Field Theory
- Curvature singularities in General Relativity
- Infinite entanglement entropy of black hole
- Difficulty in measuring metric at sub-Planckian scales

$\longrightarrow$ problem is infinitesimally short distances

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Proposal: Formulate theory in terms of discrete causal order, a *Causal Set*

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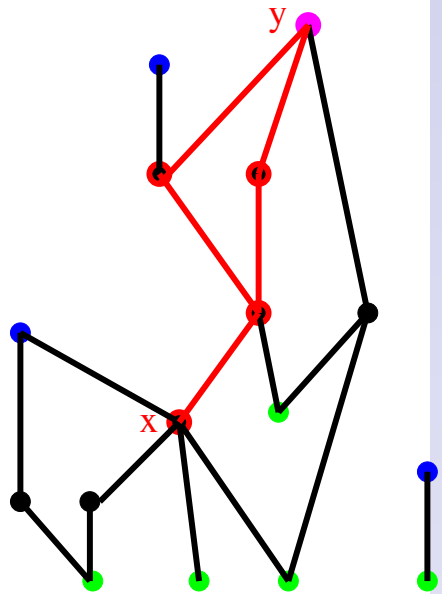
Causal Sets: Fundamentally Discrete Gravity

Based upon two main observations:

- Richness of causal structure
- Indications of some discrete substructure to spacetime



Properties of order relation $\prec$:

- irreflexive ($x \not\prec x$)
- transitive ($x \prec y$ and $y \prec z \Rightarrow x \prec z$)
- locally finite ($|\{y : x \prec y \prec z\}| < \infty$) (sometimes stronger *past finite*)

Note

- aliases: partially ordered set, poset, (partial) order, acyclic transitive directed graph, causet
- irreflexive $\Longrightarrow$ no closed causal curves
- $\exists$ reflexive (and acyclic) definition

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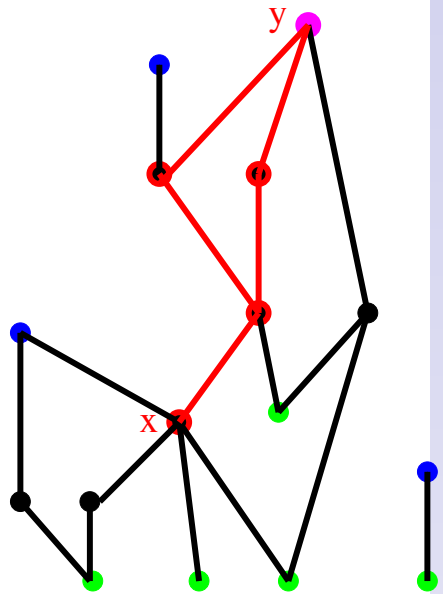

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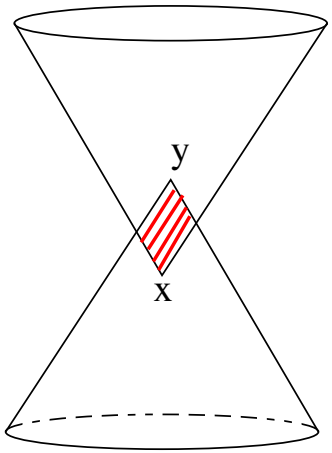
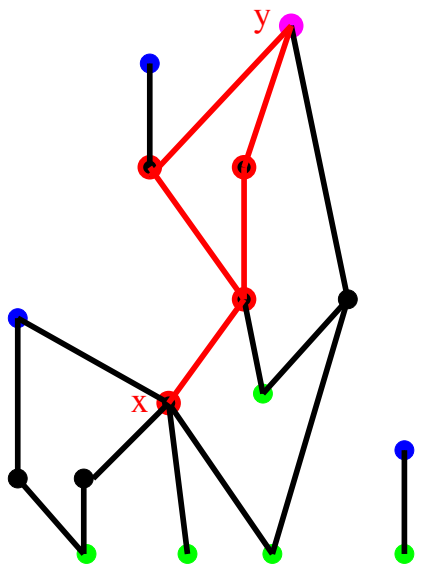
Defintions





- relation $x \prec y$
- link / “covering relation”
- ancestors & descendants
- chain
- antichain
- maximal & minimal elements
- (order) interval / Alexandrov neighborhood / Alexandrov set $[x, y] = \{z : x \prec z \prec y\}$

Spacetime Manifold as Emergent Structure

The continuum approximation

Discrete — Continuum Coorespondence:

$$\begin{array}{ccc} \text{microscopic} & \longleftrightarrow & \text{macroscopic} \\ x \prec y & \longleftrightarrow & x \in J^-(y) \\ N & \longleftrightarrow & V \end{array}$$

- *Embedding* – order preserving map $\phi : \mathcal{C} \to (M, g)$

$$x \prec y \Leftrightarrow \phi(x) \prec \phi(y) \quad \forall x, y \in \mathcal{C}$$

- *Faithful embedding ('Sprinkling')*:
  - "preserves number – volume correspondence"
  - Spacetime does not possess structure at scales smaller than discreteness scale
- $\exists$ faithful embedding $\Longrightarrow$ (M, g) *approximates* $\mathcal{C}$

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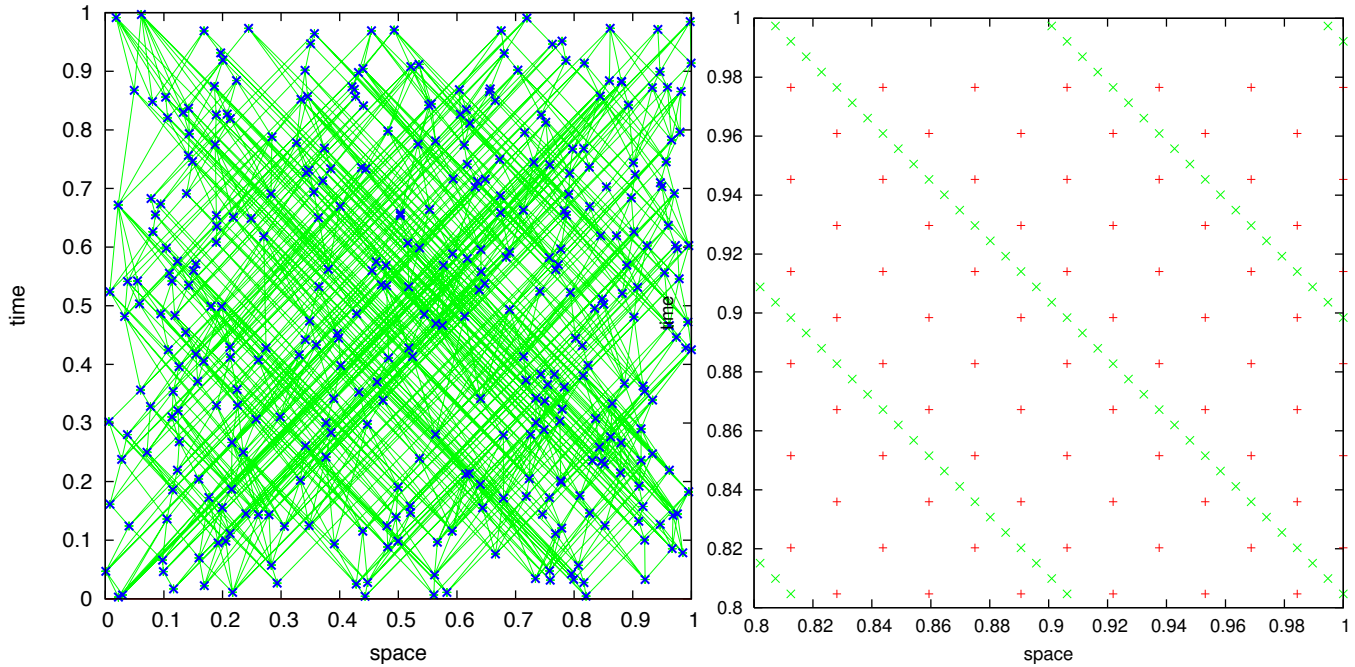
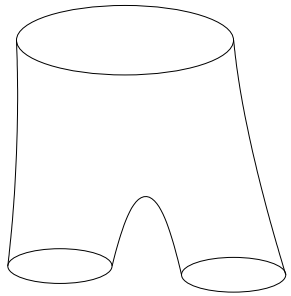
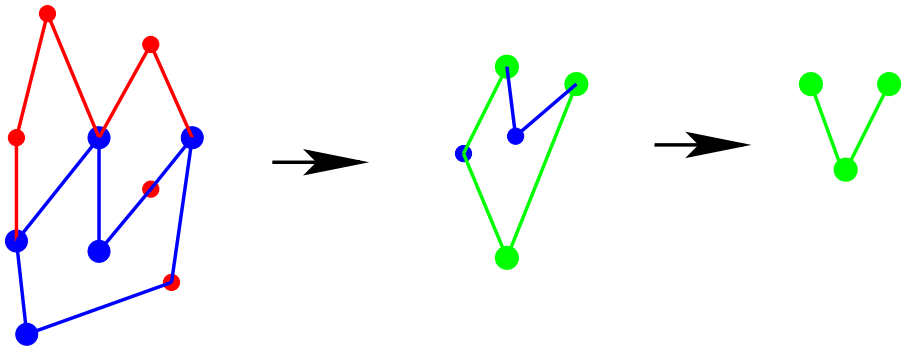


A lot from minimal structure

- discrete order $\longrightarrow$ macroscopic causal structure
- causal structure $\longrightarrow$ conformal metric
- discreteness ("N") $\longrightarrow$ volume element
- recover *all* geometry from discrete causal order
- Lorentzian signature arises naturally
- $\Longrightarrow$ unifies topology, differential structure, Lorentzian metrical geometry, causal structure



- allows topology change
- allows scale dependent topology



Dimension of Causal Sets

- Combinatoric dimension
- Causal dimension & “pixies”
- Midpoint scaling dimension
- Myrheim–Meyer dimension

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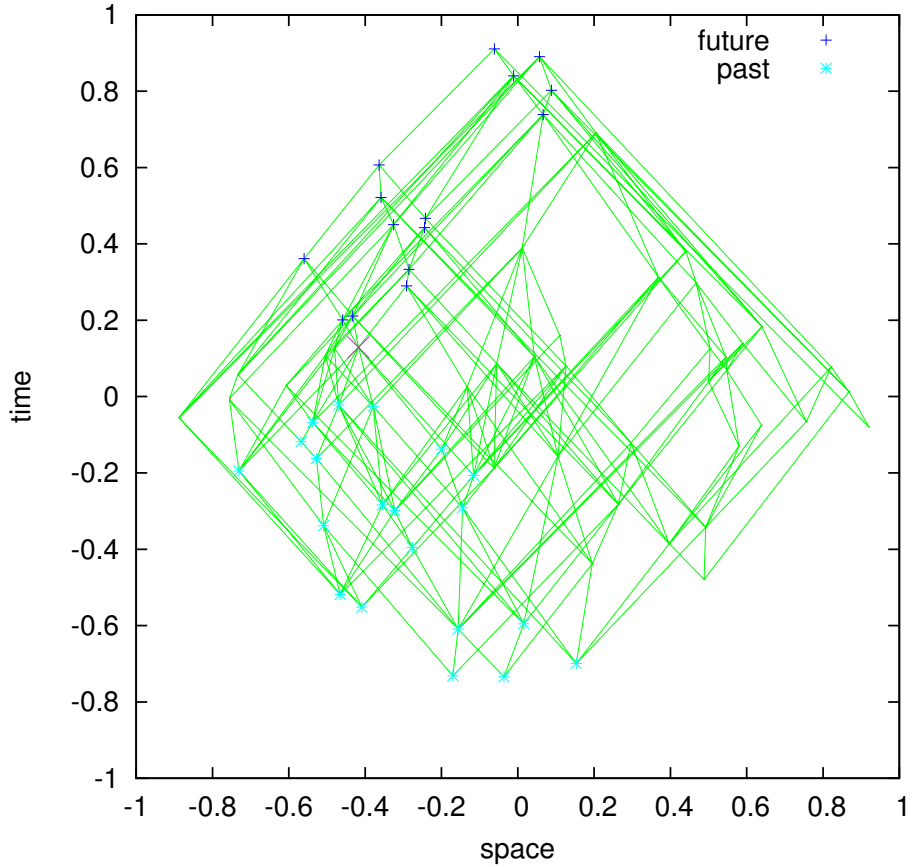
Midpoint scaling dimension

Defined on order interval $I = [a, b] = \{y | a \prec y \prec b\}$, $N = |I|$



Mid-point x maximizes $N' = \min(|[a, x]|, |[x, b]|)$

$$d \approx \log_2 \frac{N}{N'}$$

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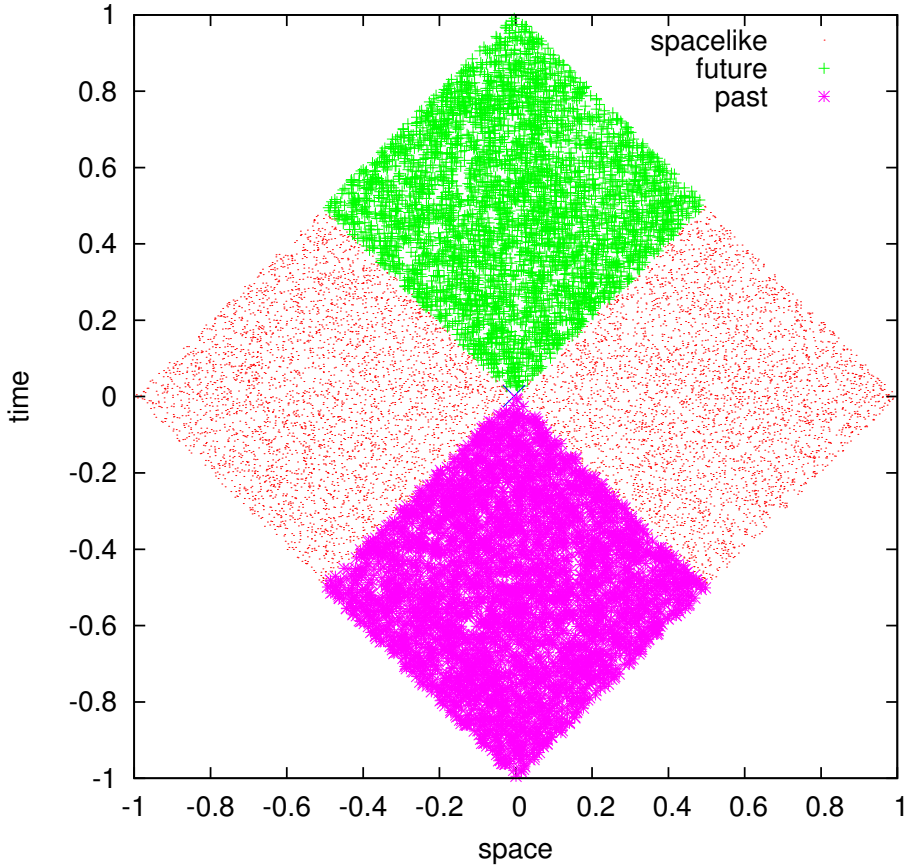

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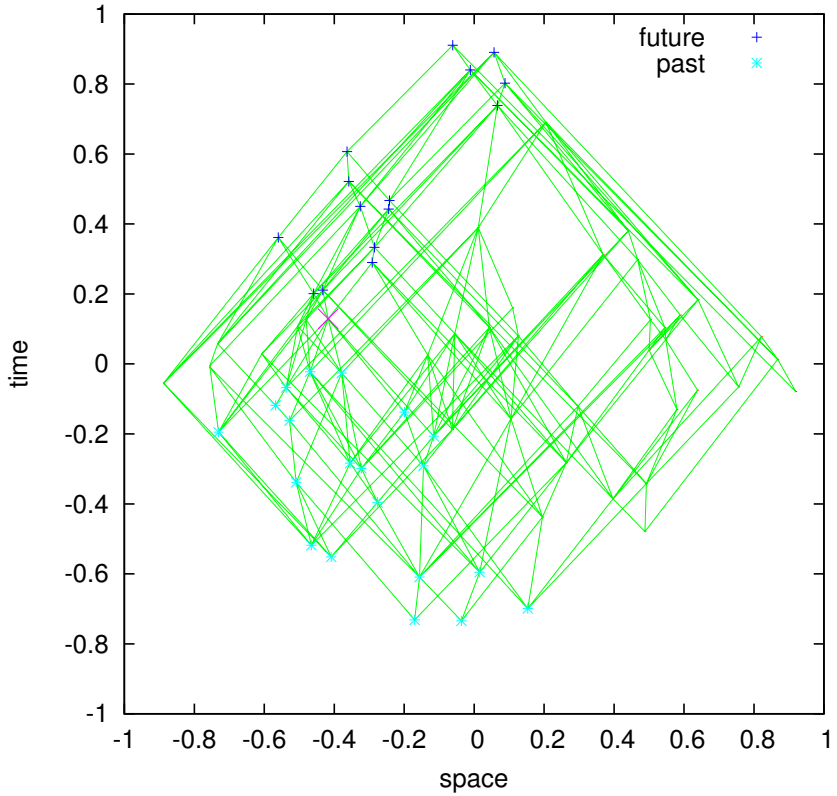

$R =$ number of relations in I (pairs x, y such that $x \prec y$)

$$f(d) = \frac{3}{2}\binom{3d/2}{d}^{-1} \qquad d \approx f^{-1}\left(\frac{R}{\binom{N}{2}}\right)$$

for $N \gg (27/16)^d$

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What do we mean by dynamics?

- Sum over histories
  Classical:

$$P(E) = \sum_{\gamma \in E} p(\gamma)$$

  Quantum:

$$A(\gamma) = e^{iS(\gamma)/\hbar}$$

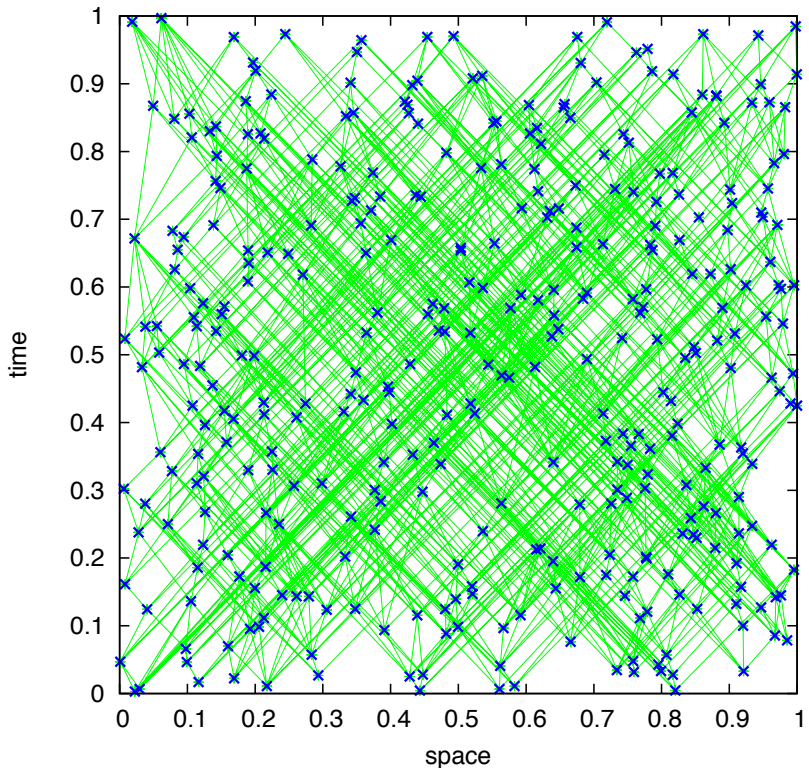
$$P(E) = \sum_{q} \left| \sum_{\gamma \in E; \gamma(T)=q} A(\gamma) \right|^2$$

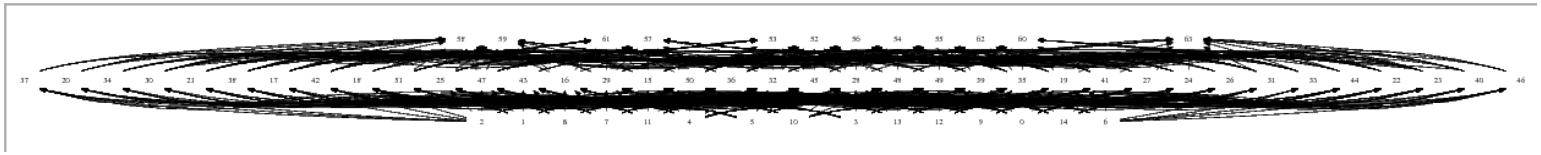
- Quantum Measure

$$P(E) = D(E, E)$$

$$D(E_1, E_2) = \text{“} \sum_{\gamma \in E_1, \gamma' \in E_2} e^{i(S(\gamma)-S(\gamma'))/\hbar} \delta(\gamma(T), \gamma'(T)) \text{”}$$

‘Entropy Crisis:’ Dynamical Emergence of the Continuum





$2^{N \ln N}$ continuua vs. $2^{N^2/4}$ Kleitman-Rothschild orders

Dynamics of Causal sets

- Discrete analog of continuum action
  - Benincasa & Dowker action: based on discrete operator which approximates $\Box - \frac{1}{2}R$
- Sequential growth dynamics (DR and Rafael Sorkin) Grow causal set, one element at a time, starting from empty set

Sequential Growth Dynamics

DR, R Sorkin

Grow causal set, 'one element at a time', beginning with empty set
Stochastic (Markov) process
Probabilities based upon three principles

- 'Internal temporality' (Causet grows only to the future)
- Discrete general covariance
- Bell causality

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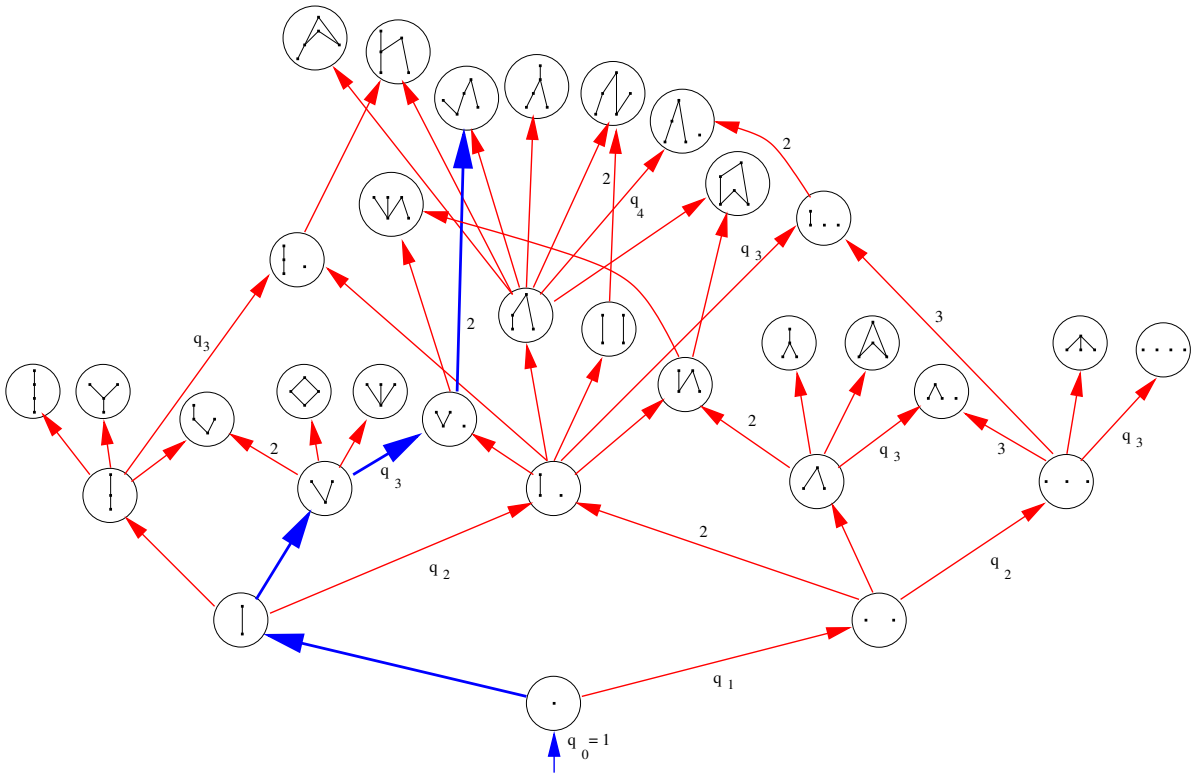

The poset of finite causal sets

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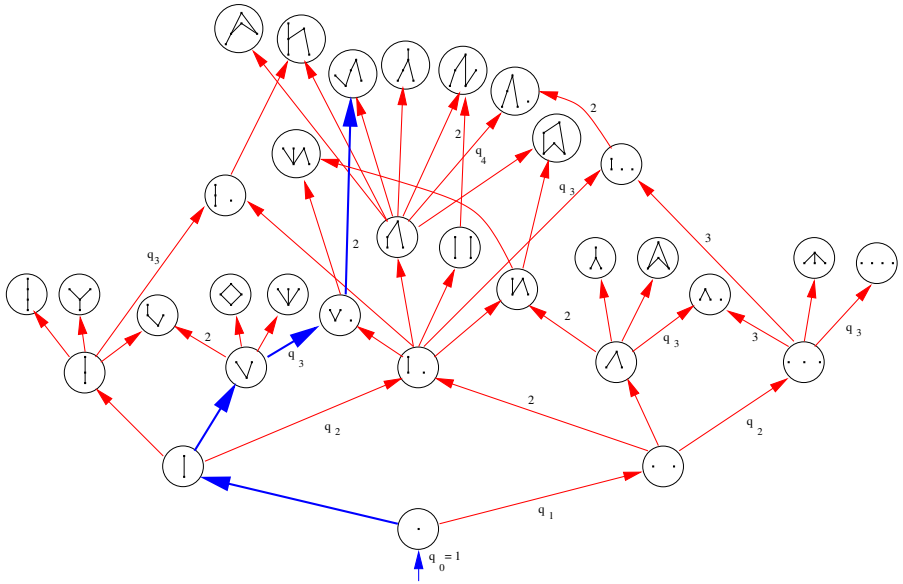

The poset of finite causal sets

$$\Pr(C_n \to C_{n+1}) \propto \sum_{k=0}^{\varpi-m} \binom{\varpi - m}{k} t_{k+m}$$

- Infinite sequence of free parameters ('coupling constants') $t_n \geq 0$
- 'Transitive percolation' dynamics $t_n = \left(\frac{p}{1-p}\right)^n$

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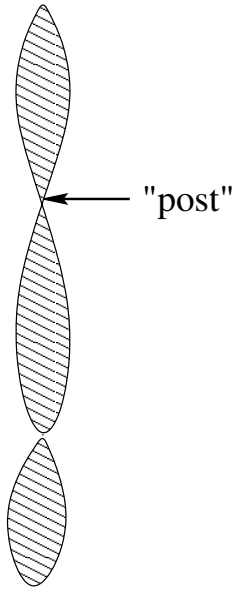
Structure of Transitive Percolation

- Completely homogeneous : future of an element is independent of anything spacelike to it
- Due to random fluctuations, however, appears inhomogeneous in time



- 'Originary' dynamics subsequent to post : Each newborn element must connect to at least one other element
- Universe expands to volume $\sim 1/p$

Originary Percolation

Random tree era

- limit $p \ll 1$
- originary — each elt chooses exactly one ancestor $\rightsquigarrow$ simple model of random tree
- exponential expansion
- future of every element itself originary percolation $\implies$ causal set is 0+1 dimensional at smallest scales
- not exactly spacetime manifold of GR

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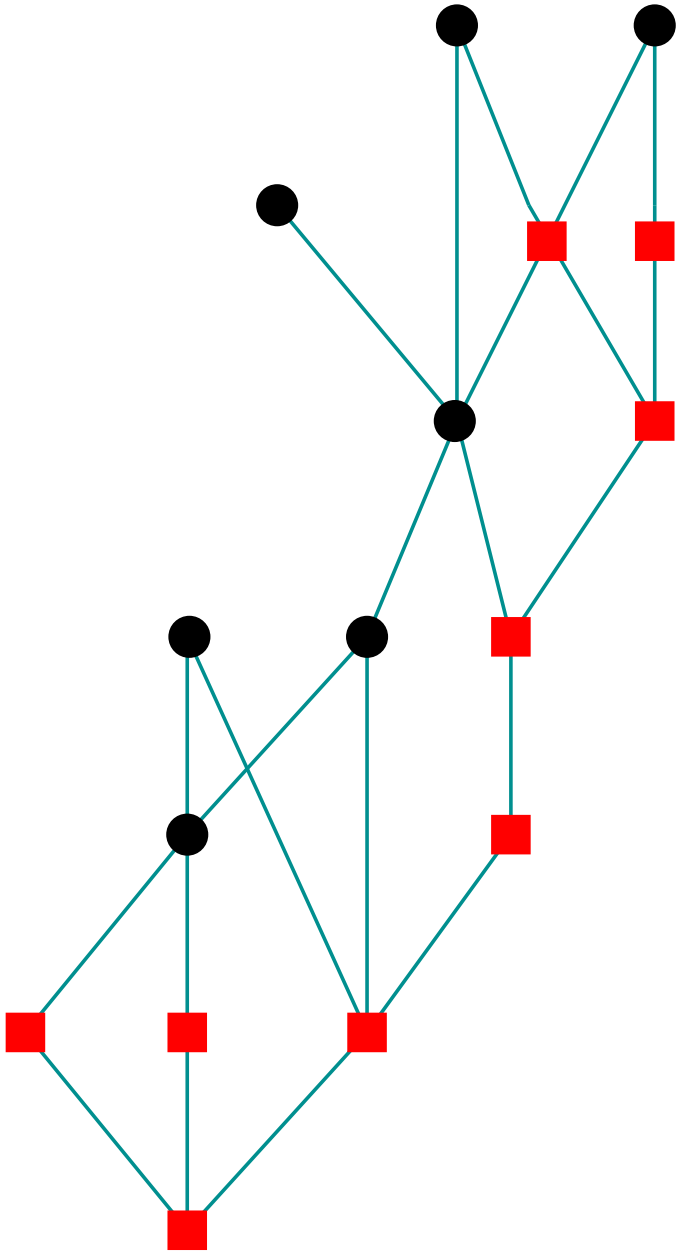

$N = 16, p = 0.2$

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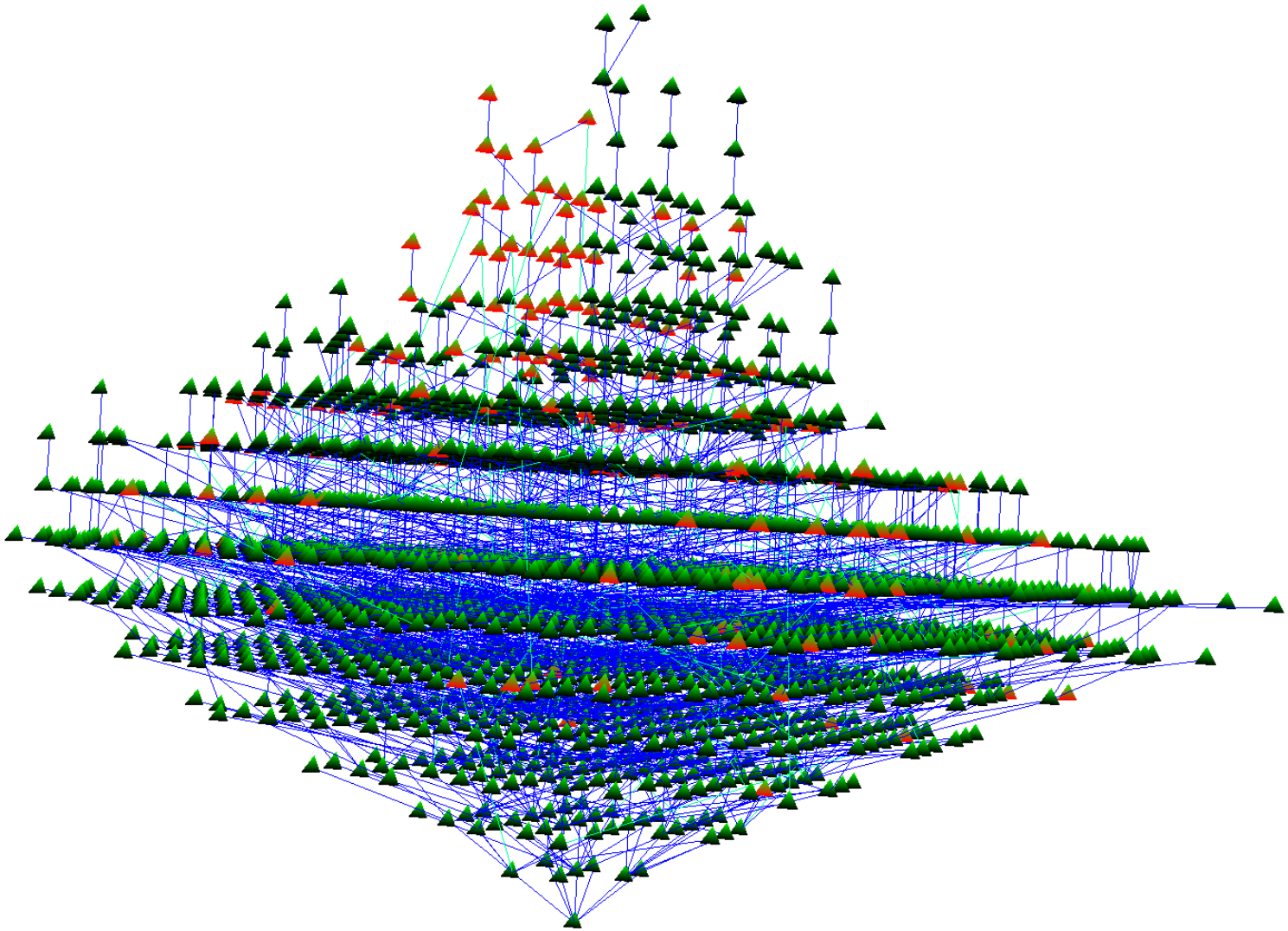

$N = 2048$, $p = 0.0001$

Intriguing Cosmological Features

- Easily evades 'entropy crisis' (DR, Sorkin)
- Tolman-Boltzmann 'cyclic' cosmology with evolving constants
  $\rightsquigarrow$ resolution to horizon and flatness problems without need for fine tuning (Sorkin et. al.)
- Evidence for de Sitter-like structure of early universe (Ahmed, DR)
- Prediction (circa 1990) $\Lambda \sim (\pm)10^{-120}$ (Sorkin)

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Dimensional reduction in quantum gravity

Carlip CQG 2017

Asymptotic safety $\Longrightarrow$ operators exhibit anomalous scaling at the UV fixed point in a way that makes the theory effectively 2d

A number of independent lines of evidence point to this:

- high temperature string theory
- causal dynamical triangulations
- loop quantum gravity
- Wheeler-DeWitt equation
- noncommutative geometry
- “broad range of other research programs”

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To what extent do we see this in causal sets?

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Evidence for dimensional reduction in causal sets

- Belenchia and Benincasa 2016
  - non-local continuum d'Alembertian derived from causal sets
  - $d \to 2$
  - based on sprinklings into Minkowski space — not directly related to gravitational dynamics
- Carlip 2015
  Myrheim-Meyer dimension of small causal sets:
  - drawn uniformly at random from causets with 4 elements
  - Kleitman-Rothschild orders
  - sprinkled into Minkowski spacetime
- First two are not terribly reasonable dynamical laws, and third falls prey to weakness in original Myrheim-Meyer dimension definition for small causets
- We examine this question in the 'full' (classical limit of the) gravitational dynamics

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- We examine this question in the 'full' (classical limit of the) gravitational dynamics

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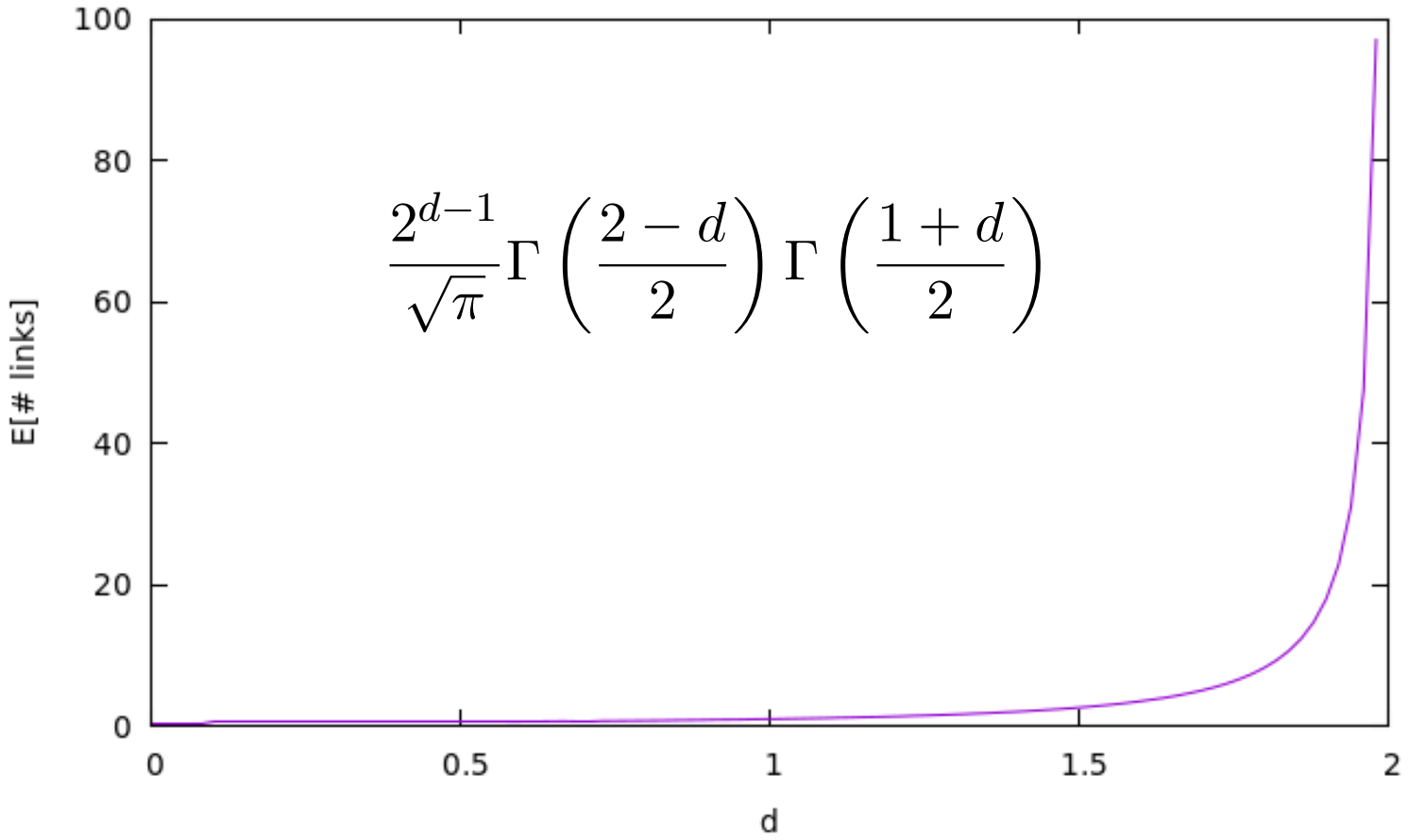
Expected number of links in $\mathbb{M}^d$



Covariant observables in cosmology

Covariant observables based on presence of '*stem sets*' (subsets which contain their own past)

in *completed histories* (for which the growth process runs to infinity)

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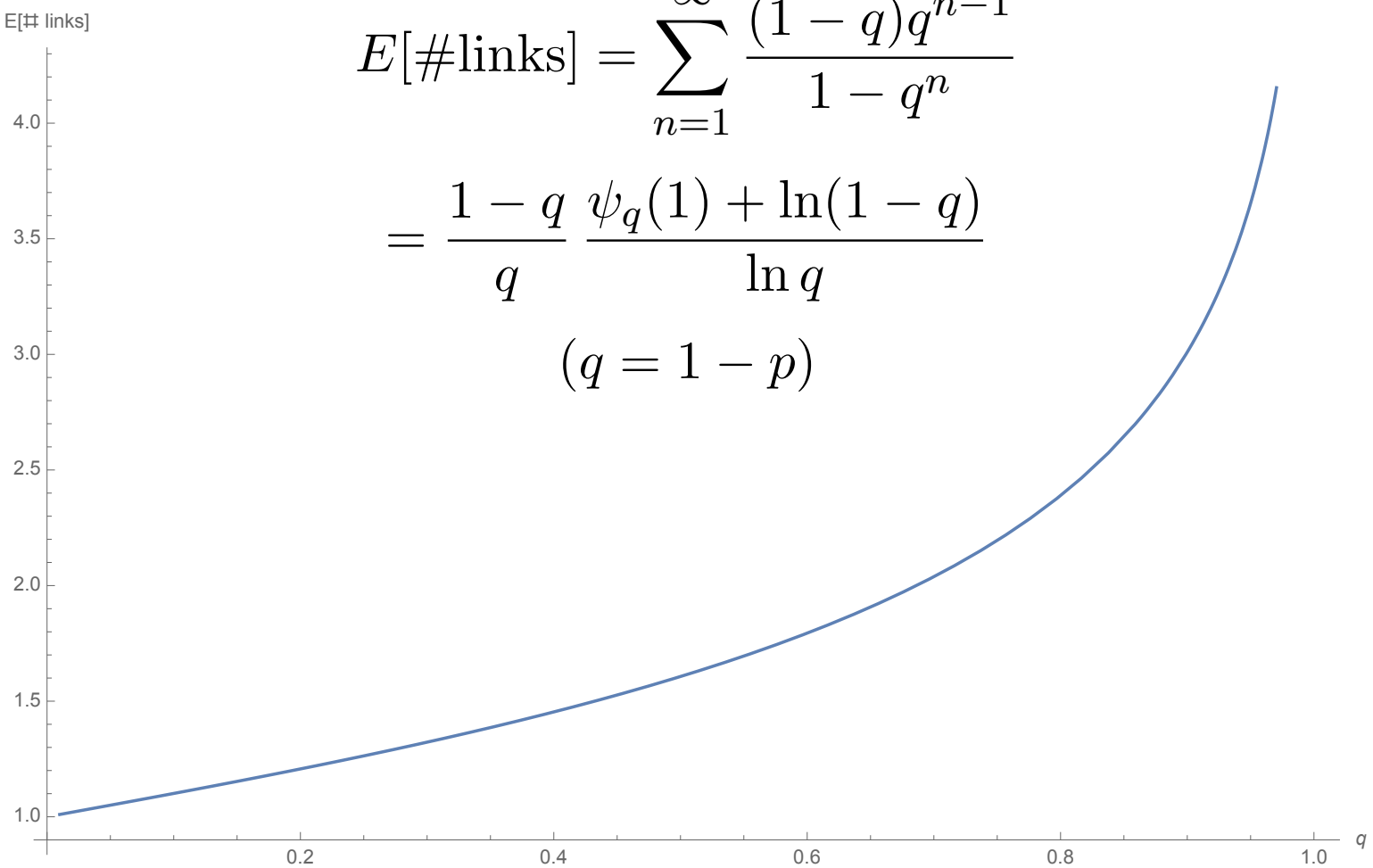
Expected number of links for originary percolation

$$E[\#\text{links}] = \sum_{n=1}^{\infty} \frac{(1-q)q^{n-1}}{1-q^n}$$

$$= \frac{1-q}{q} \frac{\psi_q(1) + \ln(1-q)}{\ln q}$$

$$(q = 1-p)$$



Causet cosmology

[R. Sorkin 2001]

For universe to expand to currently observed size:

$$p \sim 10^{-84}$$

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‘Thicker links’

- What about scaling of dimension with volume?
- ‘thicker links’ — 3-chains and larger
- Can write down double recursion for expected number of 3-chains, but difficult to convert to closed form expression
- Hard to compute further analytically – do numerically

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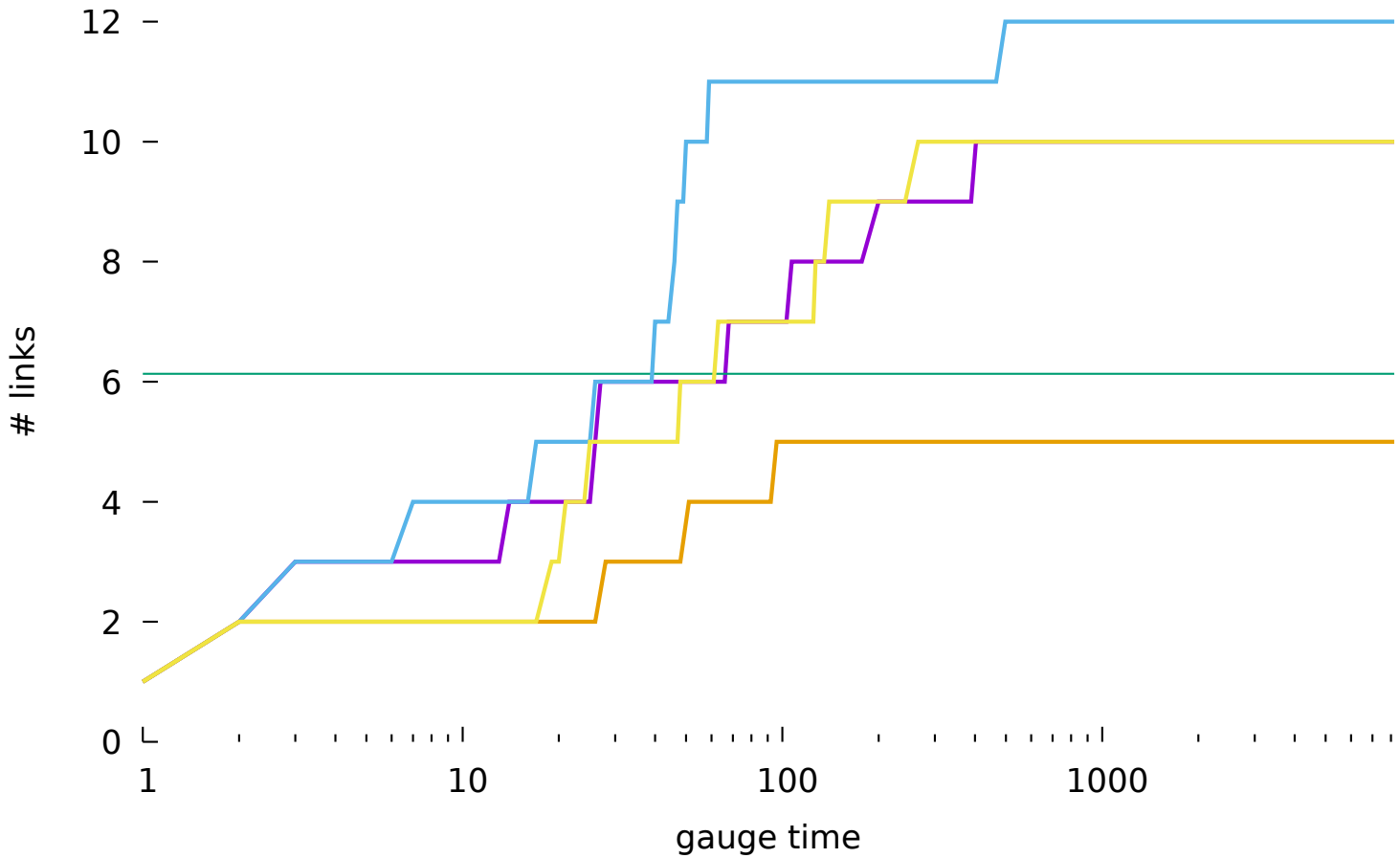
Counting ‘cosmic links’

$p = 1/256$ $N = 8192$



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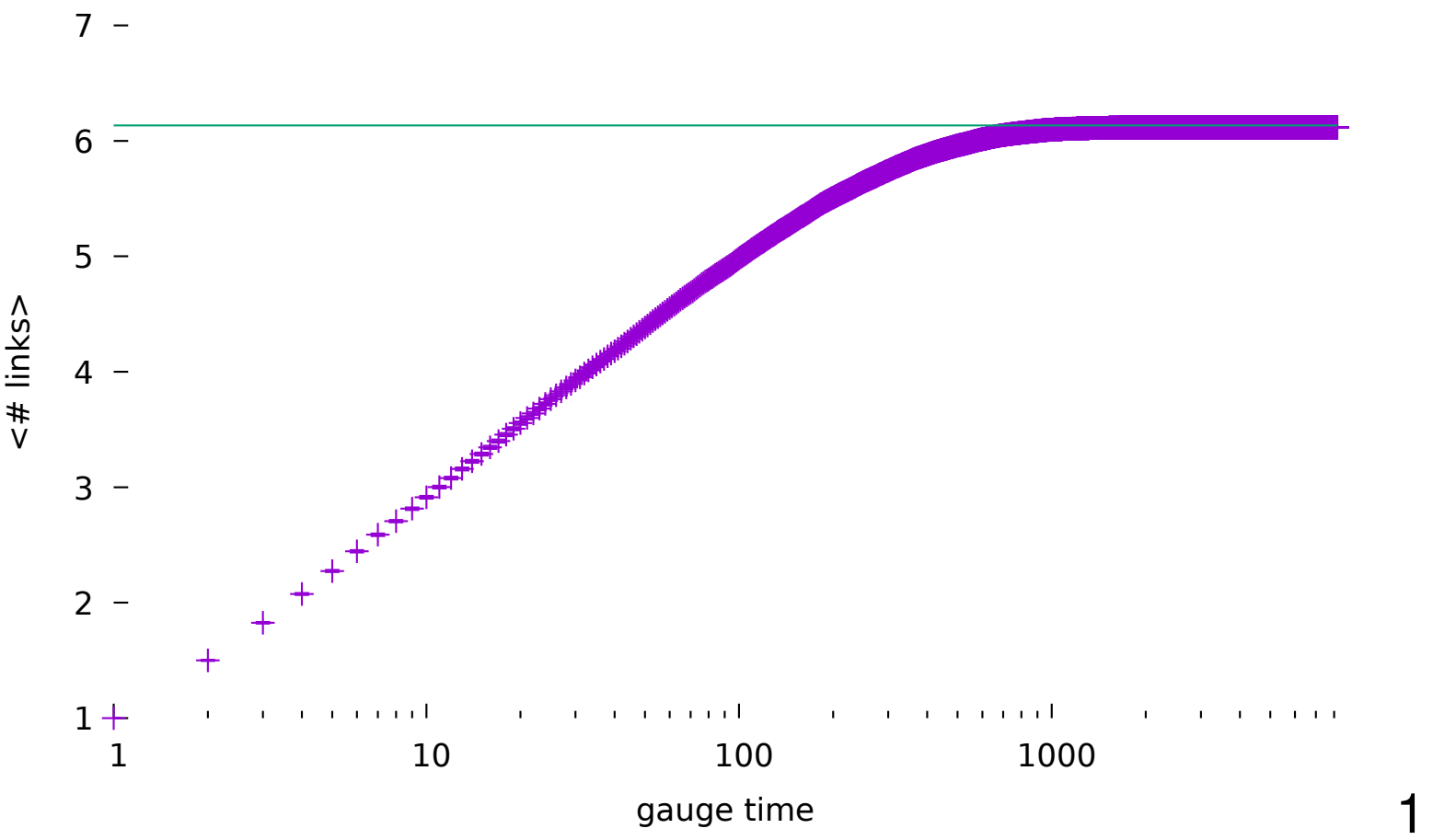
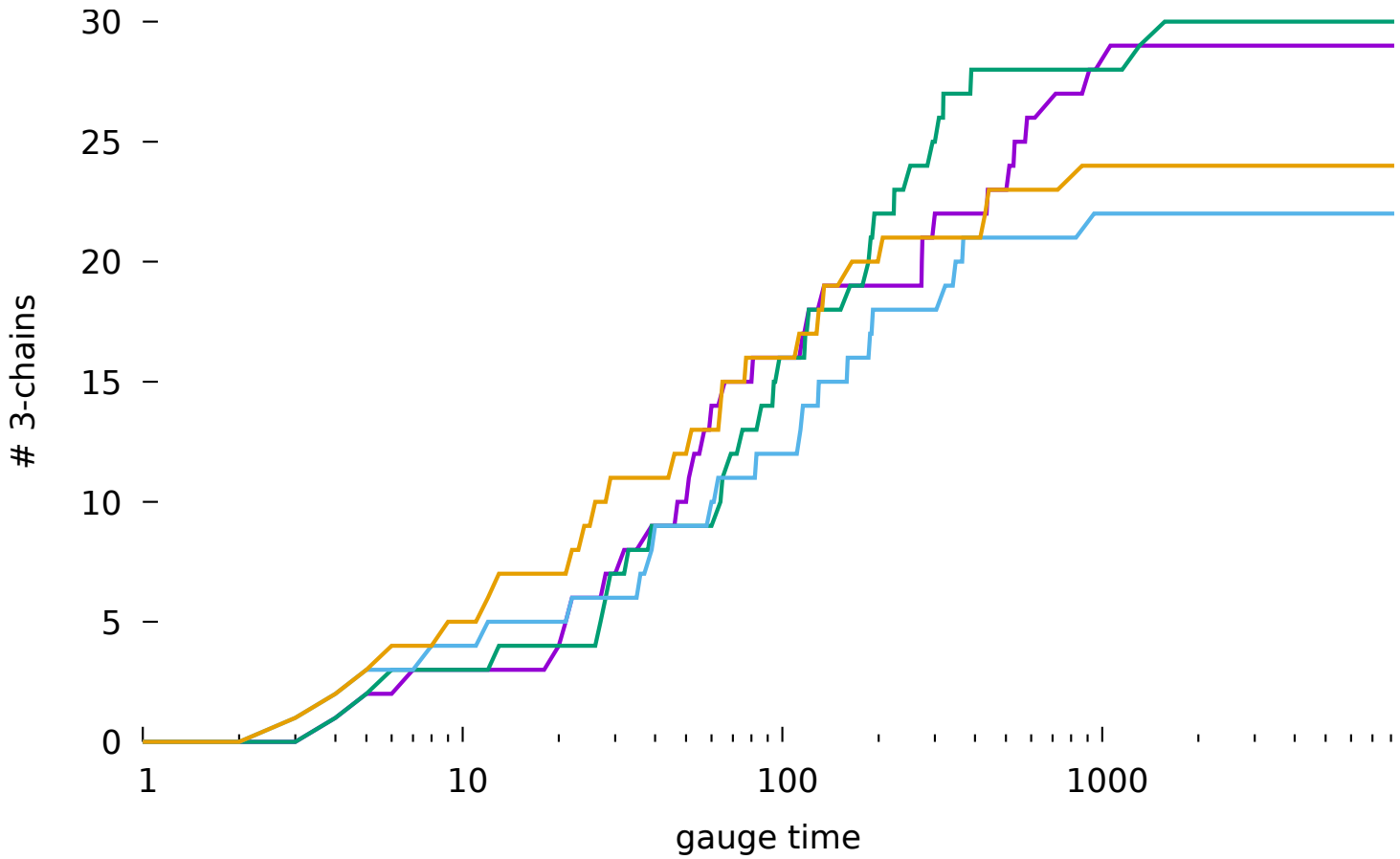
Mean number of ‘cosmic links’

$p = 1/256$ $N = 8192$ 10^4 causal sets



Counting number of cosmic 3-chains

$p = 1/256$ $N = 8192$



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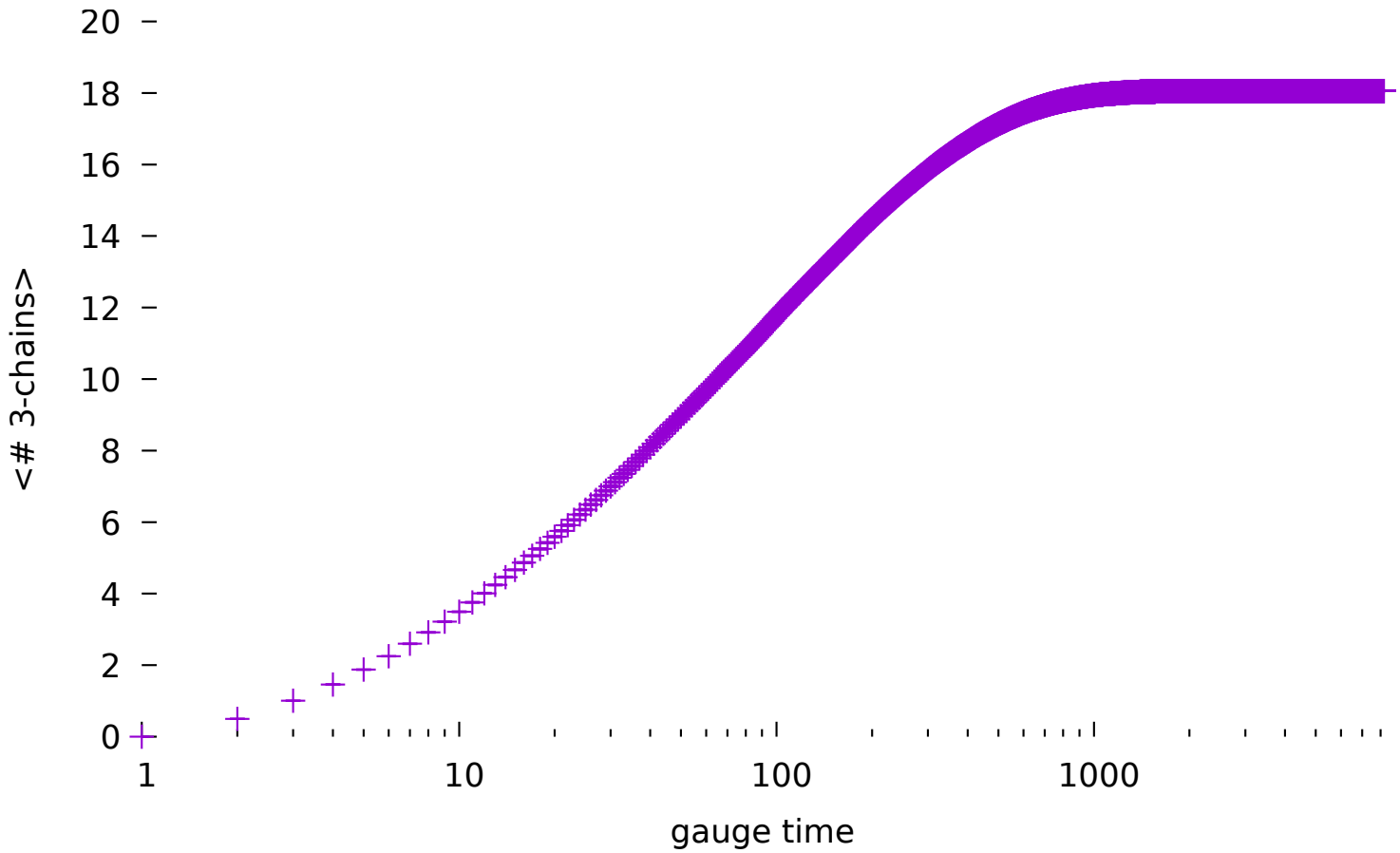
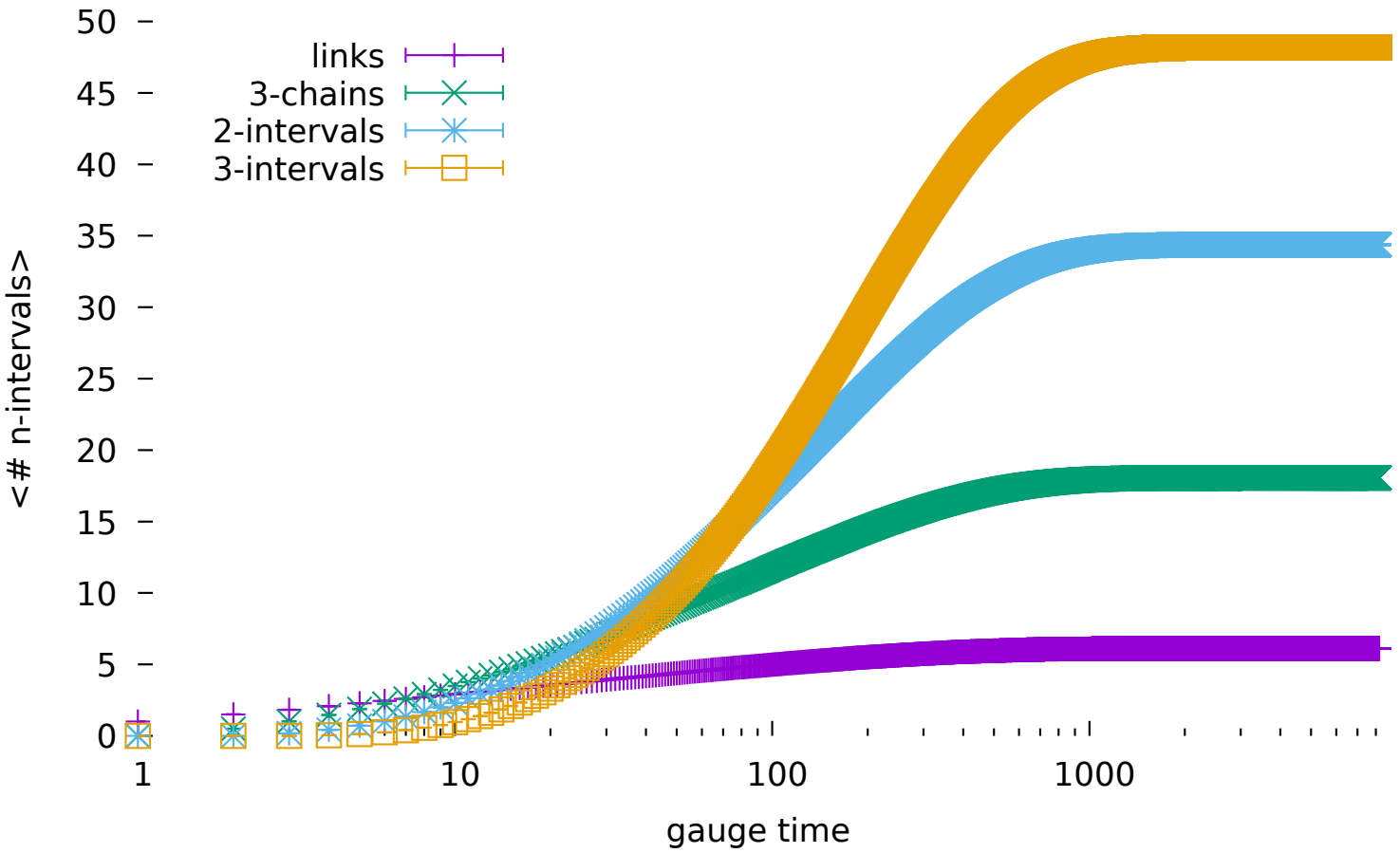


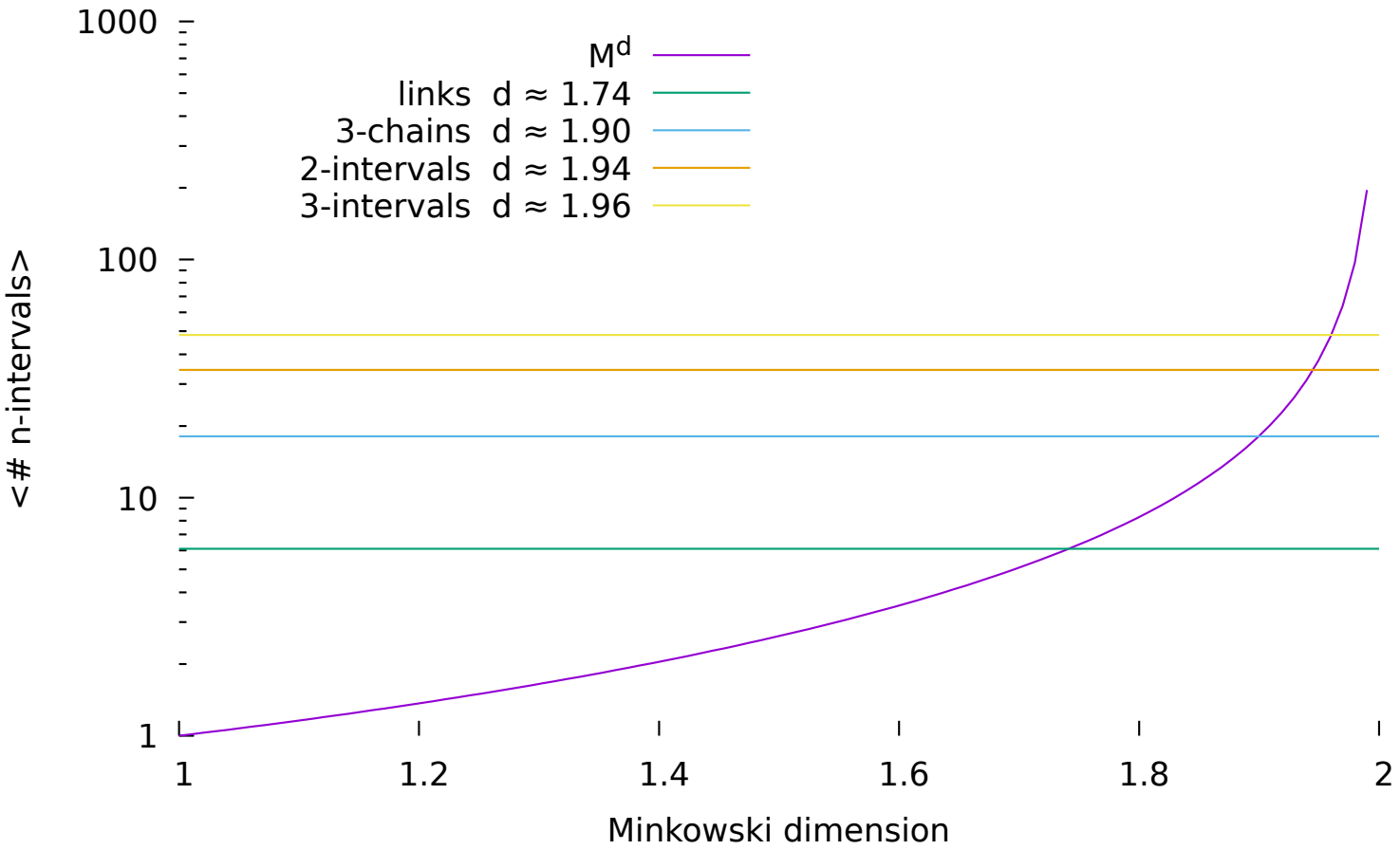
Mean number of cosmic intervals

$p = 1/256$ $N = 8192$ 10^4 causal sets



Cosmic interval counting dimension

$p = 1/256$



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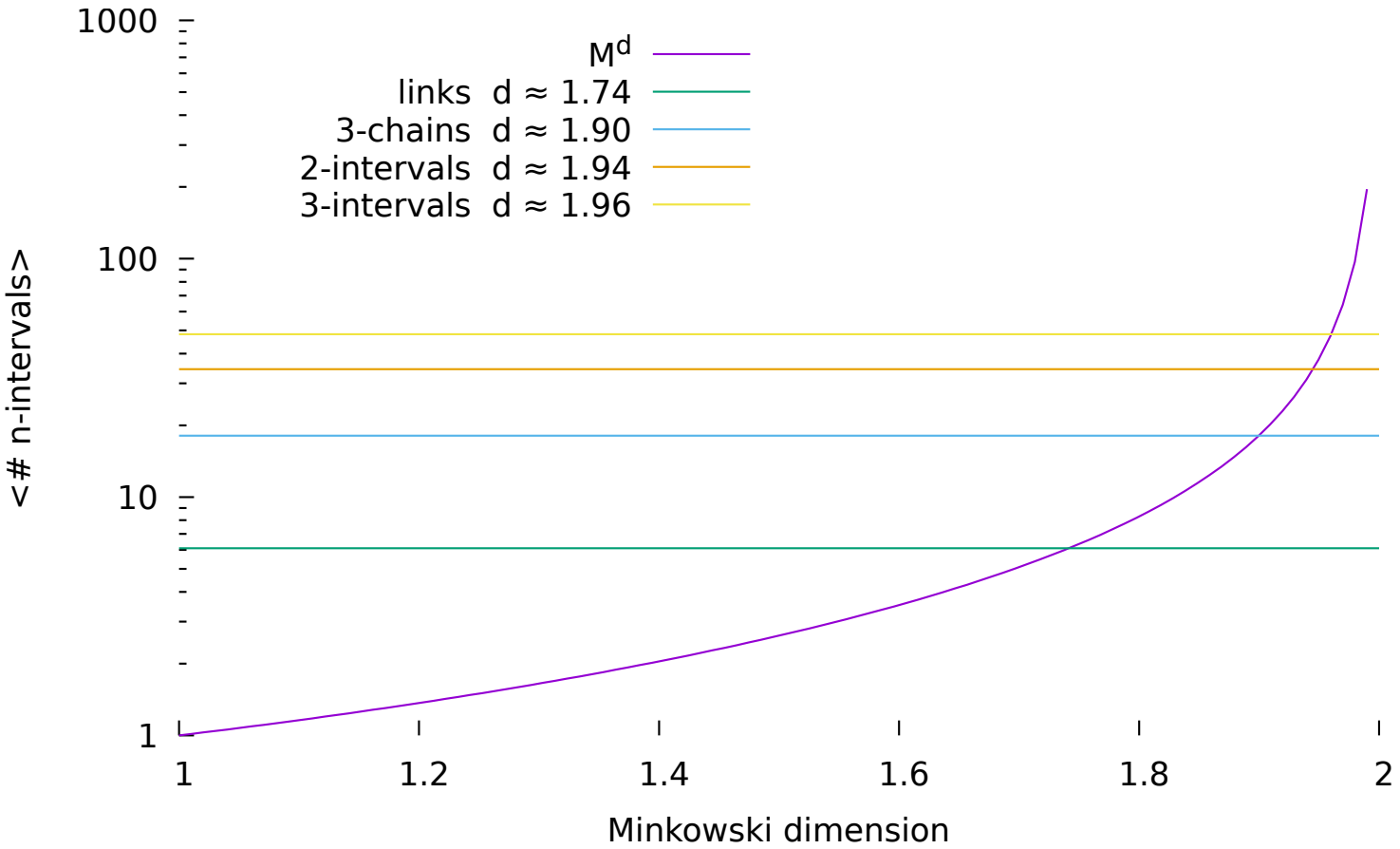

- Dimension ≈ 2 at smallest scales

Cosmic interval counting dimension

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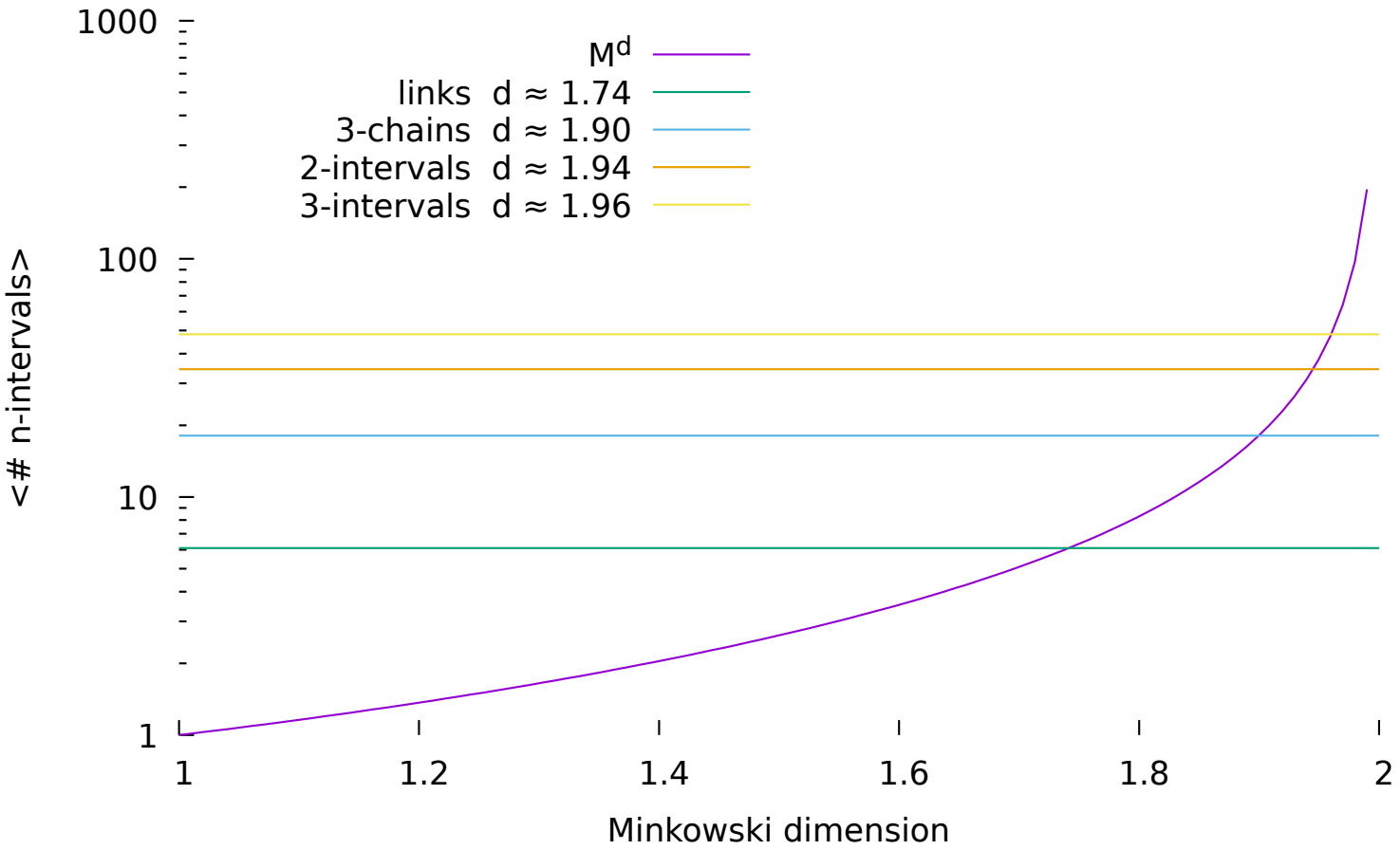

- Dimension ≈ 2 at smallest scales
- Can we ever achieve dimension > 2 ??

Interval dimension

- Consider internal structure of ‘thick’ links — what is the effective dimension of these intervals?
- Cosmic tree $\Longrightarrow$ $d = 1$ at smallest scales [Ahmed, DR PRD 2010]
- links, 3-chains surely no more than one dimensional
- What about larger intervals? Use Myrheim-Meyer dimension as simple measure of interval dimension

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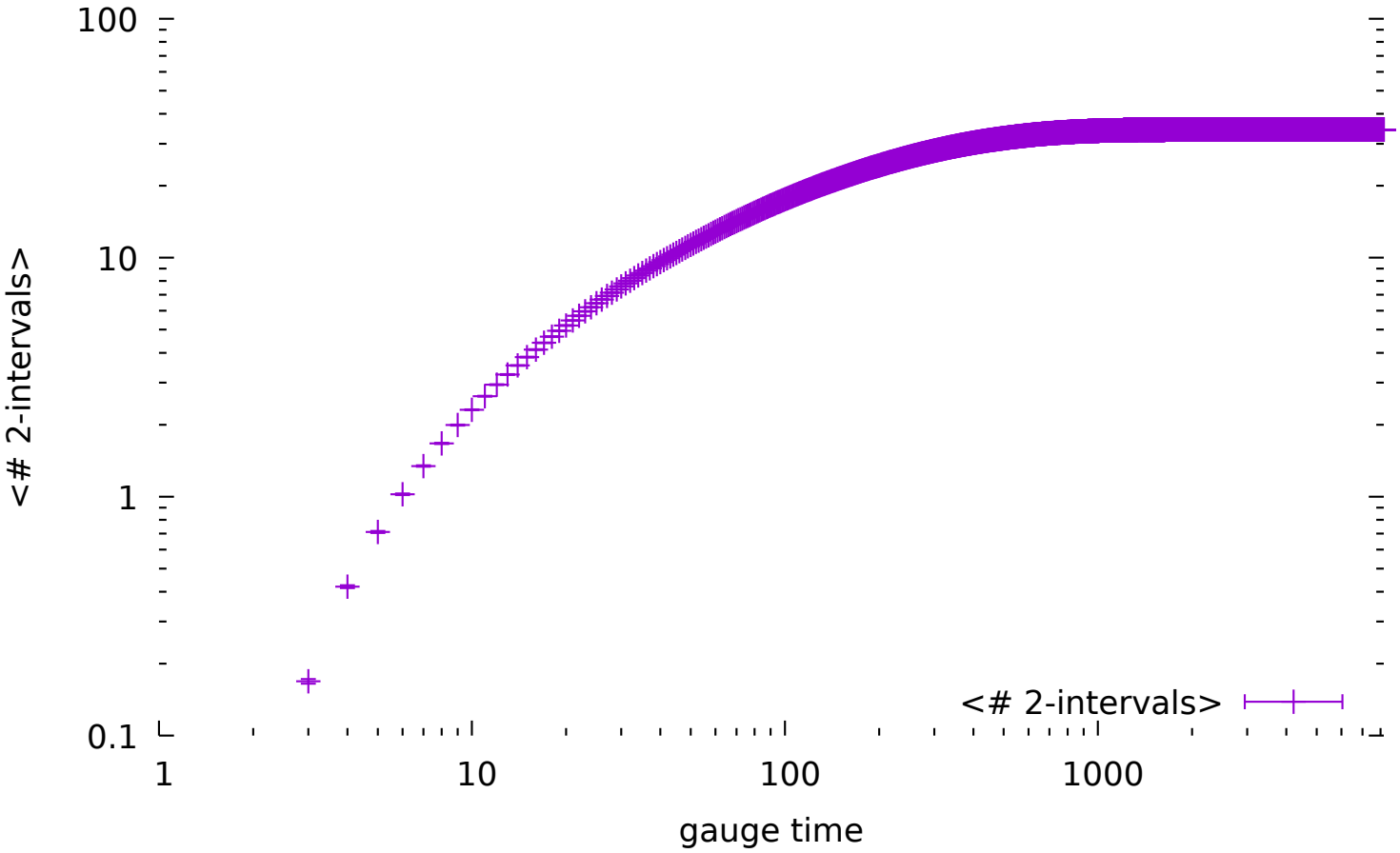
2-interval dimension

$p = 1/256$ $N = 8192$ 10^4 causal sets



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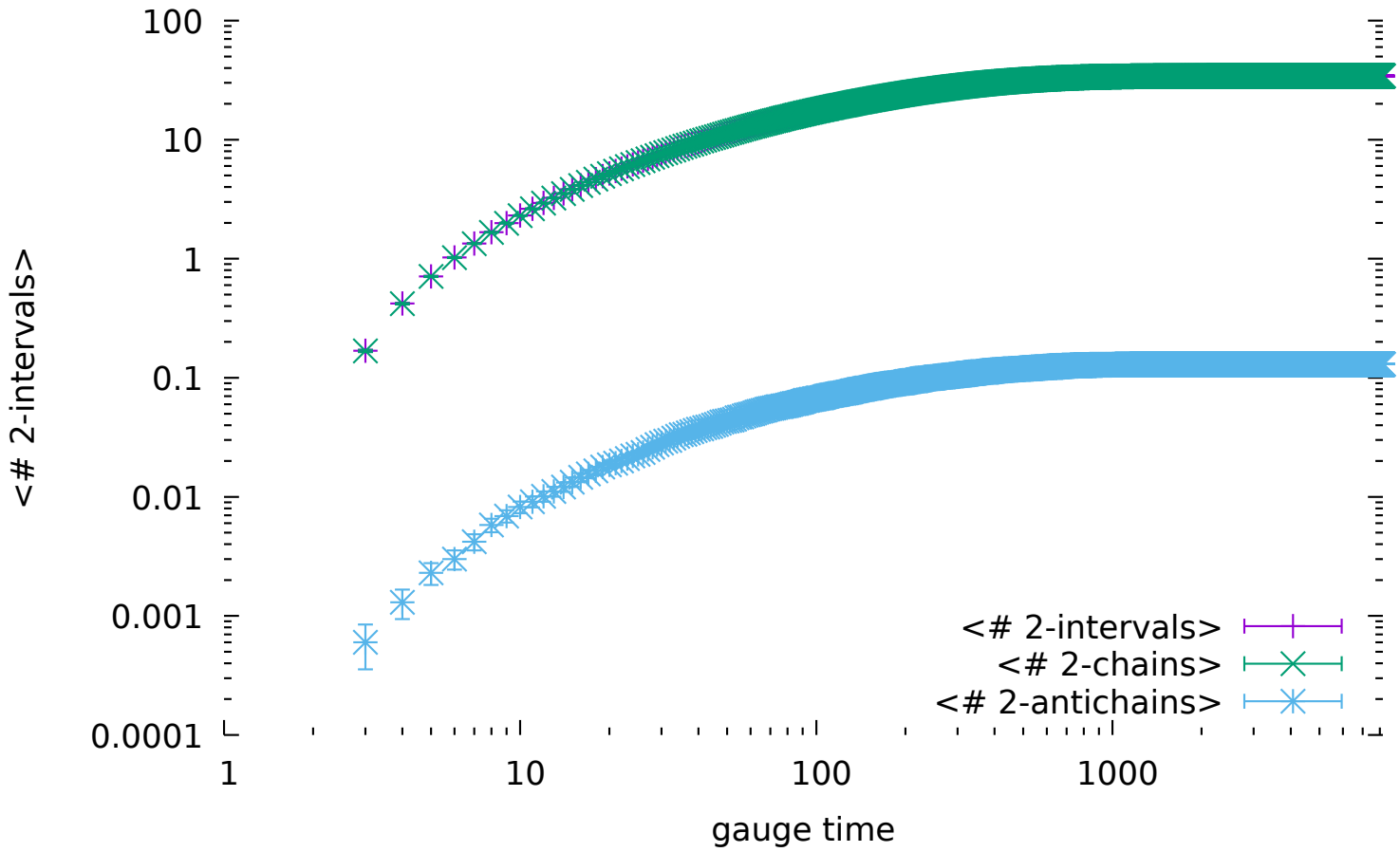

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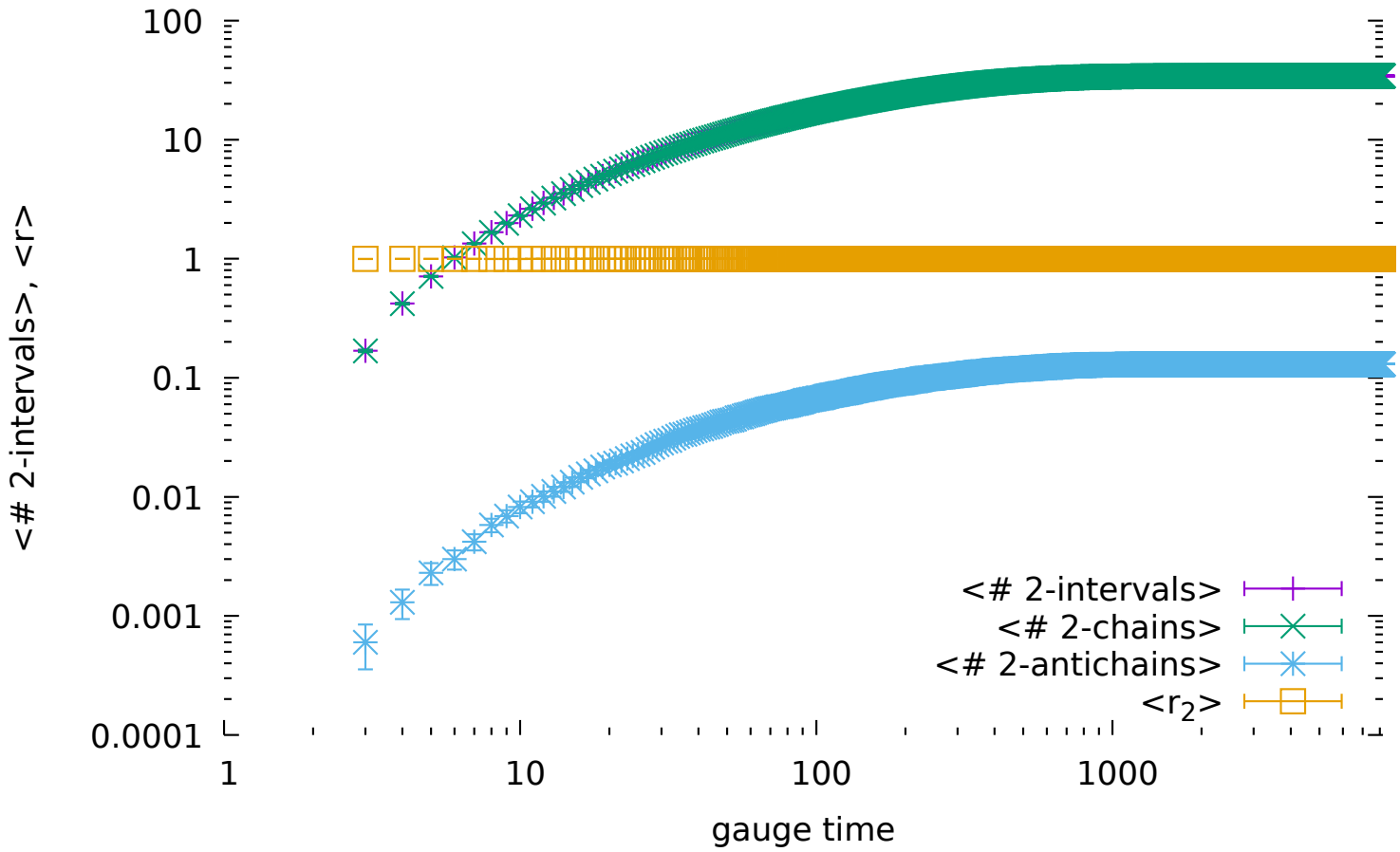

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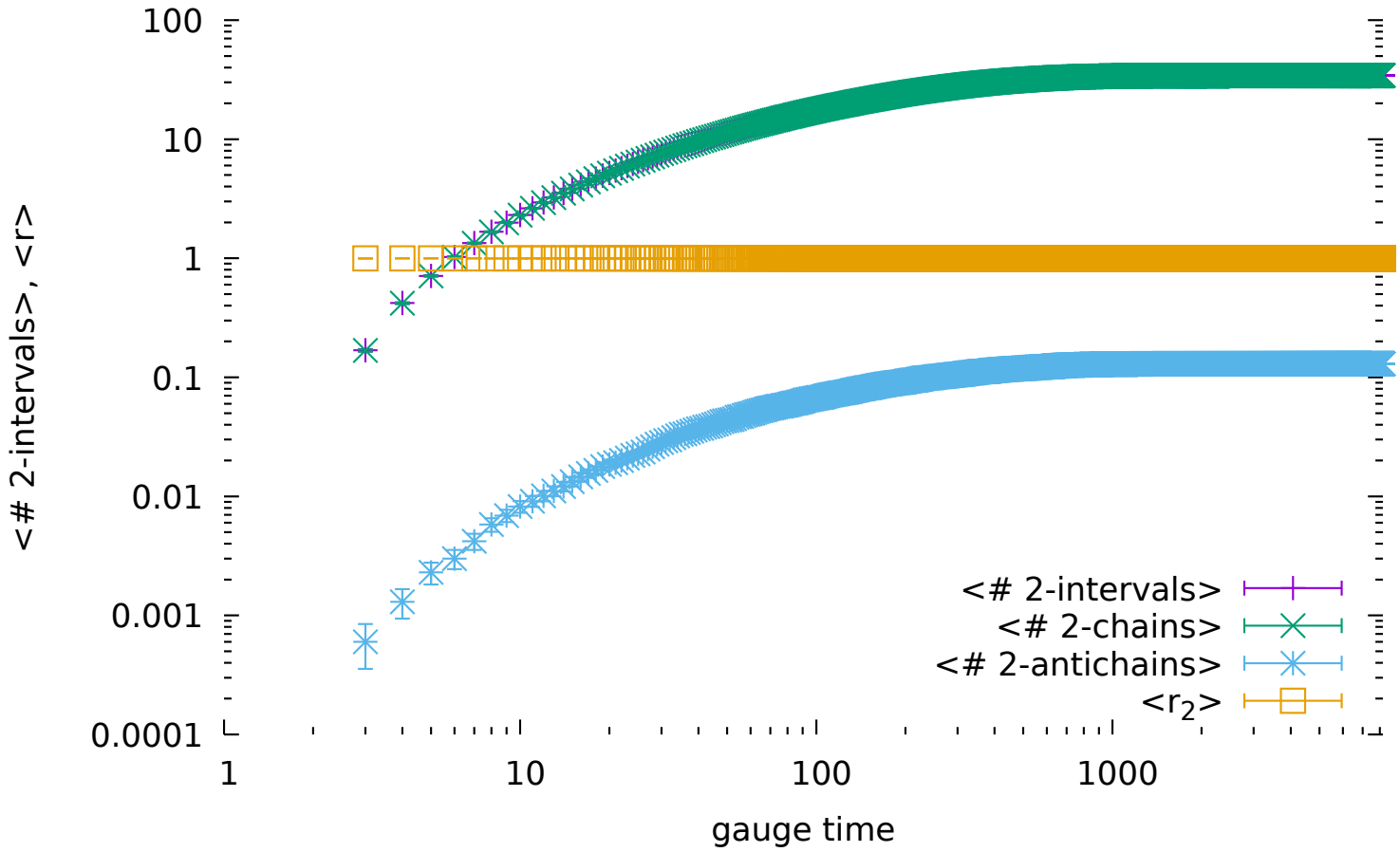

$$< r_2 >= 0.9964 \implies d \approx 1.006$$

3-interval dimension

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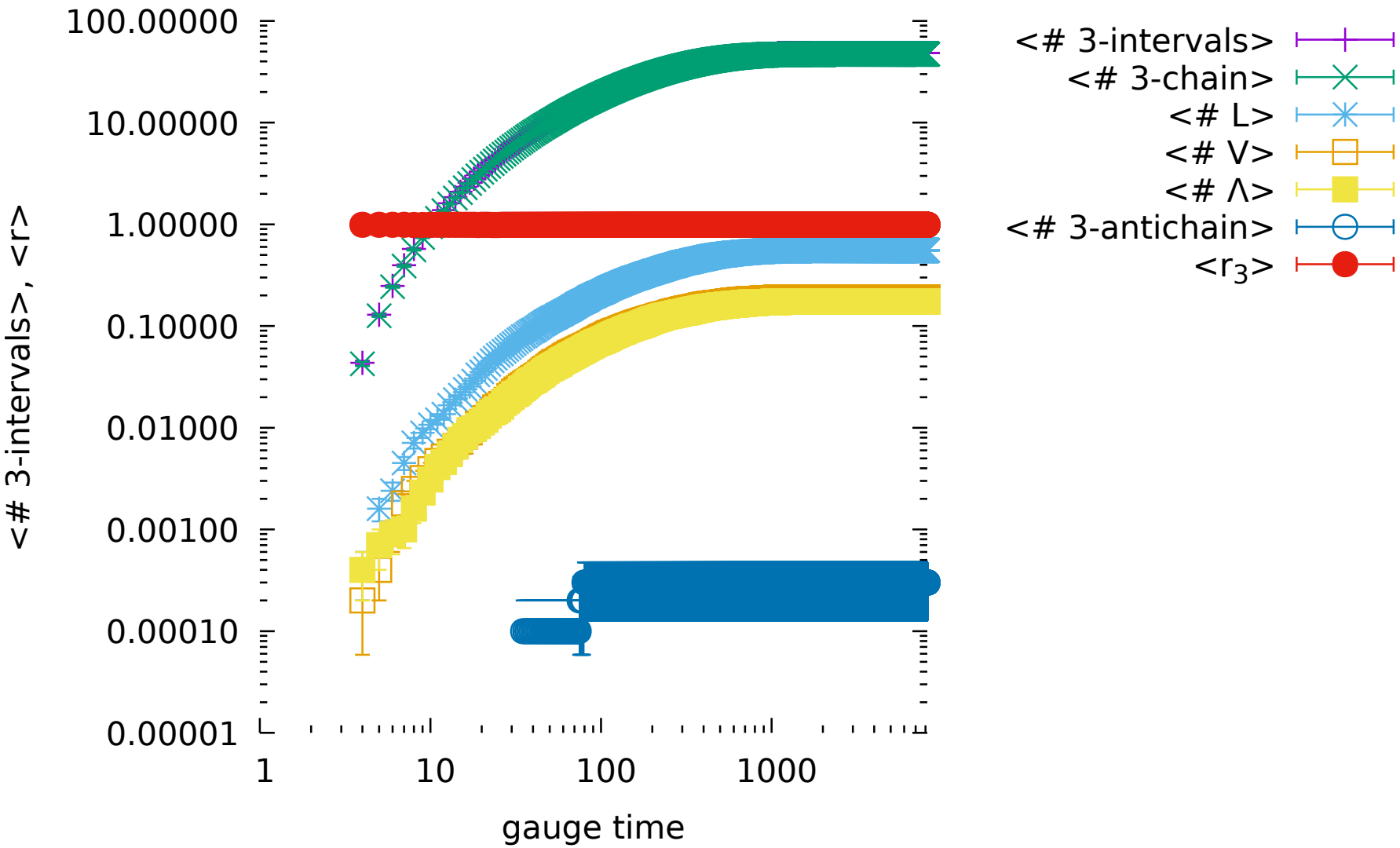

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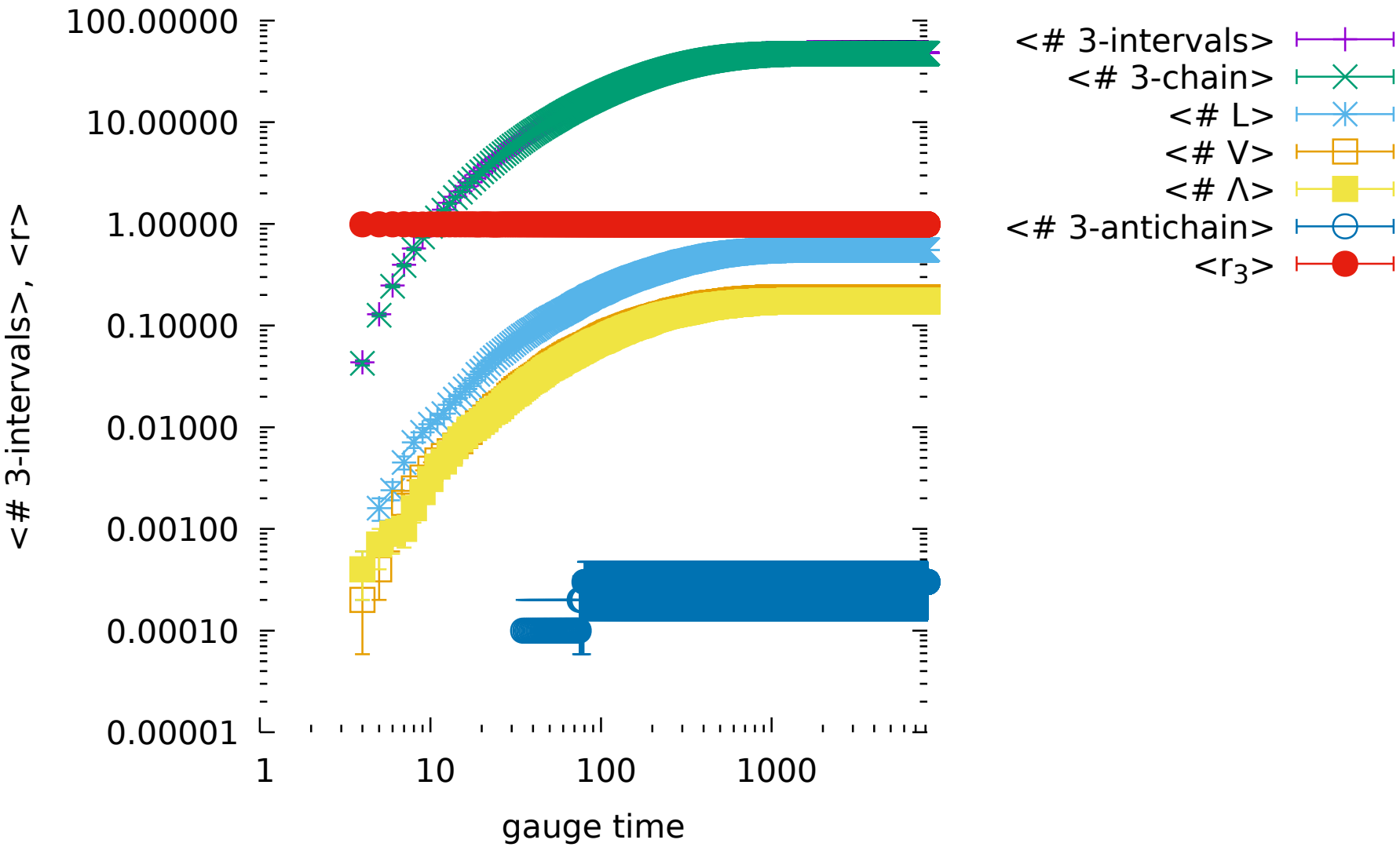
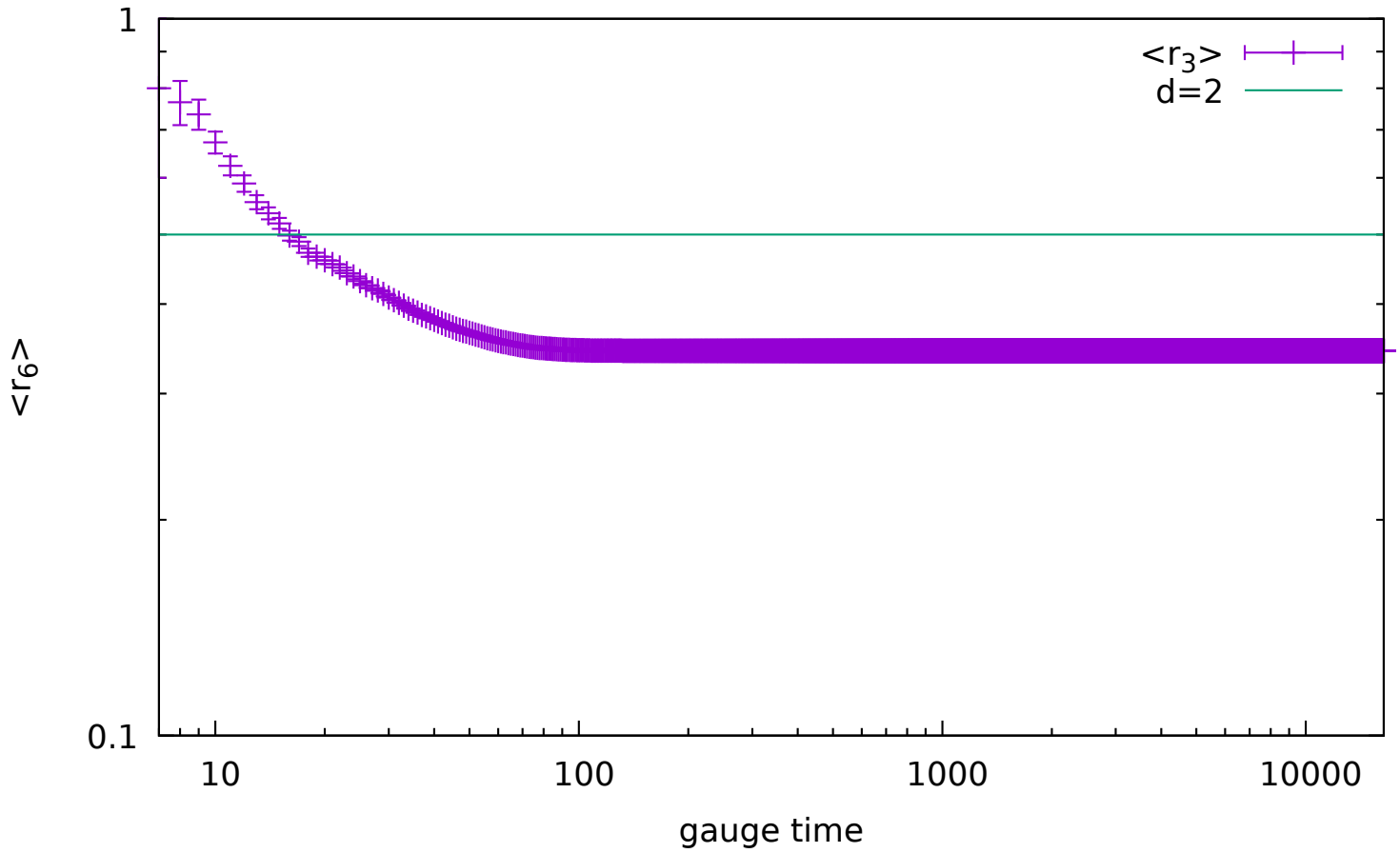


$$< r_3 >= 0.9901 \implies d \approx 1.016$$

6-interval dimension

$p = 1/256$ $N = 2^{14}$ 10^4 causal sets



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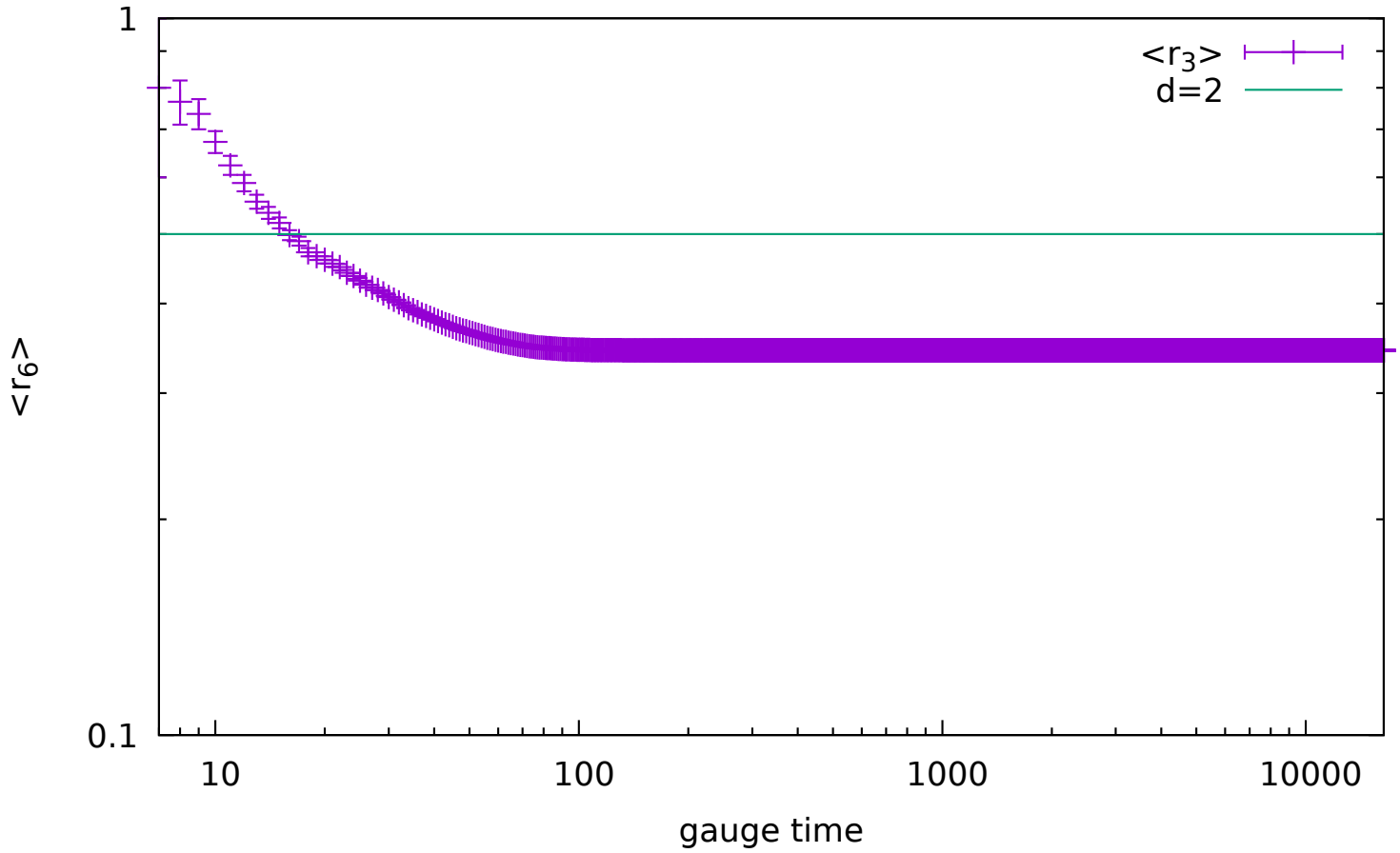

$$< r_6 >= 0.345 \implies d \approx 2.49$$

Non-manifold $\Longrightarrow$ dimension not uniform!

Two notions of dimension:

- ('thick') link counting
- dimension of 'cosmic intervals'

"perfect rose" by Liz West

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Summary and Conclusions

- Construct dynamical law consistent with fundamental discreteness, covariance and causality. Expect quantum theory to mimic classical in many ways, as it does for continuum theories.
- Causal set sequential growth dynamics gives rise to universe with many intriguing features, even if not familiar spacetime of GR.
- Link counting dimension manifests dimensional reduction to 2 at smallest scales.
- Interval dimension is 1 at smallest scales, as expected for the cosmic tree, and grows to > 2 at larger scales.
- deSitter is likely more appropriate than Minkowski $\rightsquigarrow$ May allow $d > 2$ for link counting.

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